

Might have Minkowski discovered the cause of gravity before Einstein?

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OUTLINE

We will never know how physics would have developed had Hermann Minkowski lived longer; particularly interesting is the question of whether Minkowski might have discovered the cause of gravity before Einstein.

As there is no way to reconstruct what might have happened in the period 1909-1915 I will outline here what steps had been logically available to Minkowski on the basis of his results - his three papers on relativity.

Then I will briefly discuss whether the implications of these results would lead to the modern theory of gravitation – Einstein's general relativity.

On the one hand, Einstein's way of thinking based on conceptual analyses and thought experiments now seems to be the only way powerful enough to decode the unimaginable nature of gravitation.

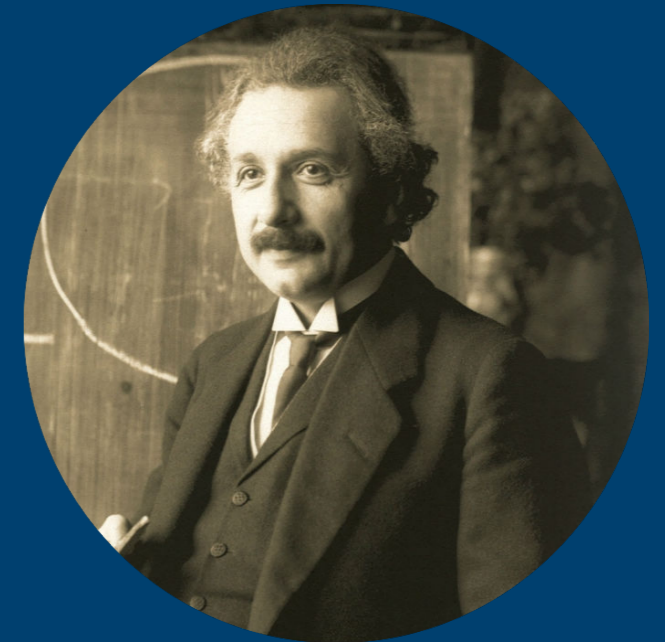
However, on the other hand, after Minkowski had written his three papers on relativity, he (had he lived longer) and his friend David Hilbert might have formed an unbeatable team in theoretical physics and might have discovered general relativity (surely under another name) before Einstein.



H. Minkowski



In 1907 Einstein had already been well-ahead of Minkowski when he made a gigantic step towards the new theory of gravity:



“I was sitting in a chair in the patent office at Bern when all of a sudden a thought occurred to me: “If a person falls freely he will not feel his own weight.” I was startled. This simple thought made a deep impression on me. It impelled me toward a theory of gravitation.”

Einstein had been so impressed by this insight that he called it the “happiest thought” of his life. And indeed this is a crucial point – Einstein first realized that **no gravitational force acted on a falling body.**

I think in order to understand better what Minkowski could have done, had he lived longer, it is important first to give the two indications of why he appears to have realized **independently**

- the equivalence of the times of observers in relative motion and
- that the Lorentz transformations can be regarded as rotations in 4D space

This happened almost certainly as late as the summer of 1905 before Minkowski read Einstein's 1905 (*Annalen der Physik* received the paper on June 30, 1905) and before reading Poincare's longer paper "*Sur la dynamique de l'électron*" which appeared in 1906.

There are two indications of that which cannot be merely ignored.

First, Max Born's two recollections quoted in the next two slides; there is no reason whatsoever to suspect that Born would invent such recollections.

Second, what is far more important, however, is the full-blown four-dimensional formalism Minkowski reported on December 21, 1907 and the depth of his understanding of the electrodynamics of moving bodies and the absolute four-dimensional world; such a revolution in both physics and mathematics could not have been possible if he had merely developed others' ideas.

Minkowski's involvement with the electrodynamics of moving bodies began in the summer of 1905 when he and his friend David Hilbert co-directed a seminar in Göttingen on the electron theory (dealing with the electrodynamics of moving bodies). Einstein's paper on special relativity was not published at that time; *Annalen der Physik* received the paper on June 30, 1905. Poincaré's longer paper "*Sur la dynamique de l'électron*" was not published either; it appeared in 1906. Also, "Lorentz's 1904 paper (with a form of the transformations now bearing his name) was not on the syllabus".

Minkowski's student Max Born, who attended the seminar in 1905, recalled in 1959 what Minkowski had said during the seminar:

I remember that Minkowski occasionally alluded to the fact that he was engaged with the Lorentz transformations, and that he was on the track of new interrelationships.

Again Born wrote in his autobiography about what he had heard from Minkowski after Minkowski's lecture "Space and Time" given on September 21, 1908:

He told me later that it came to him as a great shock when Einstein published his paper in which the equivalence of the different local times of observers moving relative to each other were pronounced; for he had reached the same conclusions independently but did not publish them because he wished first to work out the mathematical structure in all its splendour. He never made a priority claim and always gave Einstein his full share in the great discovery.

Here I will stress particularly the core of general relativity which reflects Einstein's "happiest thought" – the geodesic hypothesis / principle / law according to which a falling particle is not subject to a gravitational force. The geodesic hypothesis is regarded as "a natural generalization of Newton's first law," that is, as "a mere extension of Galileo's law of inertia to curved spacetime."

According to the geodesic hypothesis in general relativity the worldline of a free particle is a timelike geodesic in spacetime.

This means that in general relativity a particle, whose worldline is geodesic, is a free particle which does not "feel" its acceleration (i.e. does not resist its acceleration). In other words, such a particle moves by inertia.

The geodesic hypothesis has been confirmed by the experimental fact that particles falling toward the Earth's surface **offer no resistance** to their fall.

This experimental fact that particles do not resist their fall (i.e. their apparent acceleration) means that they move by inertia and therefore no gravitational force is causing their fall. It should be emphasized that a gravitational force would be required to accelerate particles downwards only if the particles resisted their acceleration, because only then a gravitational force would be needed to overcome that resistance.

In his famous lecture *Space and Time* Minkowski outlined his profound idea of regarding physics as spacetime geometry:

“The whole world presents itself as resolved into such worldlines, and I want to say in advance, that in my understanding the laws of physics can find their most complete expression as interrelations between these worldlines.”

Then he explained the difference between inertial motion (represented by a straight worldline) and accelerated motion (represented by a curved or rather **deformed** worldline) and remarked: “Especially the concept of acceleration acquires a sharply prominent character.”

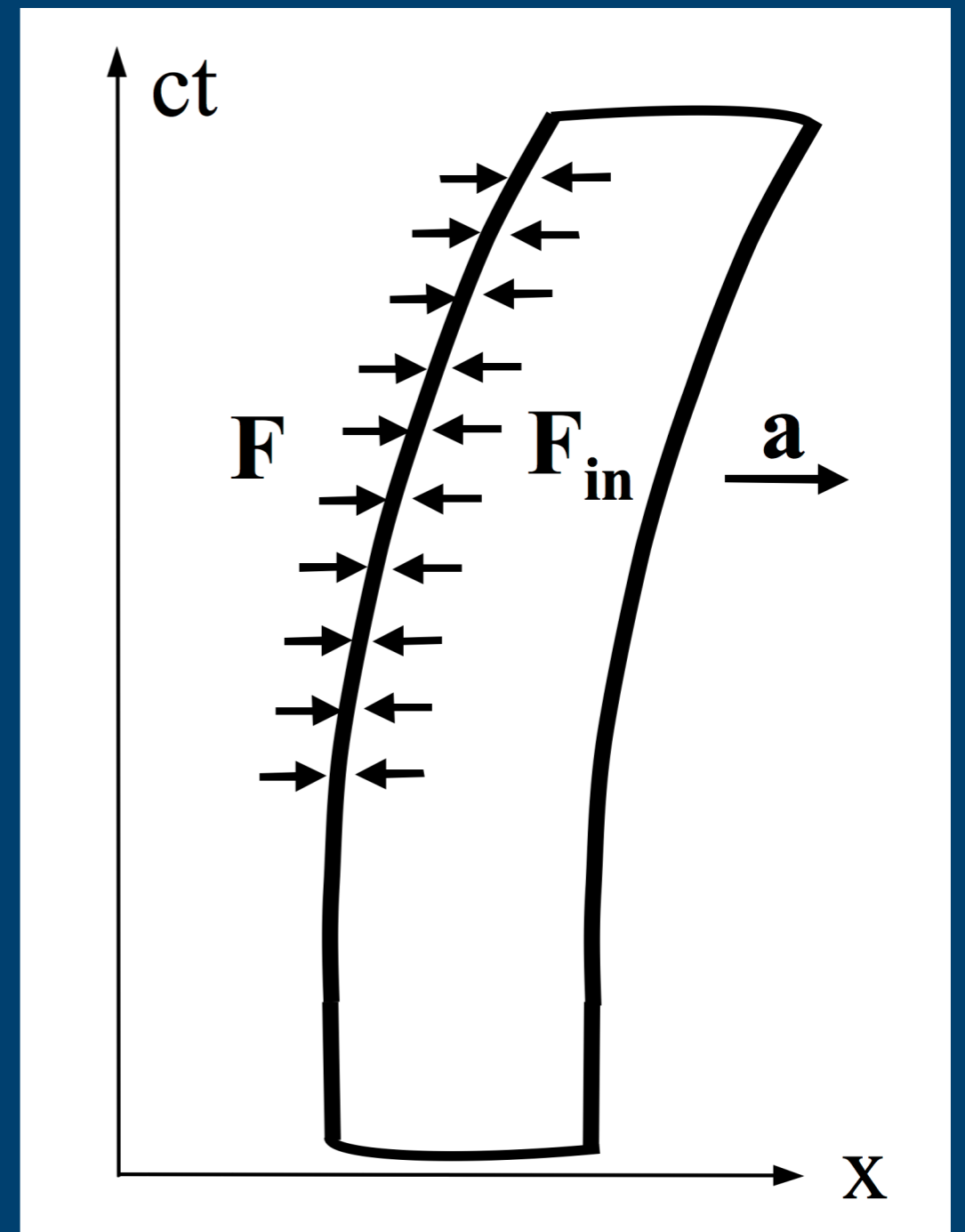
As Minkowski knew that a particle moving by inertia offers no resistance to its motion with constant velocity (which explains why inertial motion cannot be detected experimentally), whereas the accelerated motion of a particle can be discovered experimentally since the particle resists its acceleration, he might have very probably linked the sharp physical distinction between inertial (non-resistant) and accelerated (resistant) motion with the sharp geometrical distinction between inertial and accelerated motion (represented by straight and curved / deformed worldlines, respectively).

Then Minkowski would have had many logical possibilities to implement his program of geometrization of physics. For example, absolute acceleration is a manifestation of the absolute geometrical feature (**deformation**) of the worldline of an accelerating particle and does not imply some absolute space with respect to which the particle accelerates. As an accelerating particle is represented by a curved (deformed) worldline Minkowski might have realized that inertia – the resistance a particle offers to its acceleration – could be regarded as arising from a four-dimensional stress⁶ in the deformed worldline, or rather worldtube, of an accelerating particle.

The worldtube is real!

I will be glad to repeat Minkowski's argument and stress what Minkowski had apparently regarded as obvious - that if the worldtube were not real, length contraction would not exist.

In other words, I will repeat Minkowski's arguments that the World (i.e. spacetime) is indeed a glorious entity.



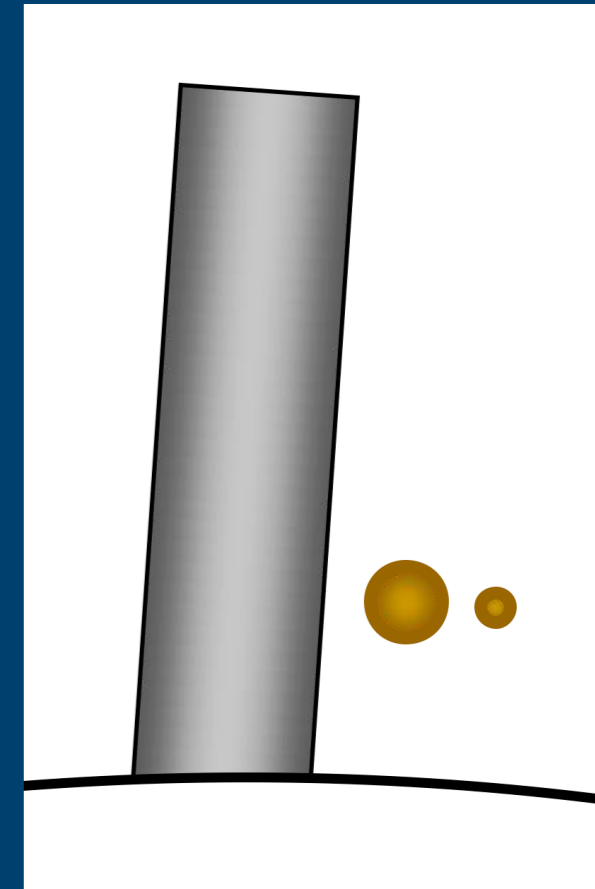
To demonstrate the enormous potential of Minkowski's criteria for inertial and accelerated motion I will discuss two scenarios in the talk.

The first assumes that Minkowski had read Galileo's works, particularly Galileo's analysis demonstrating that heavy and light bodies fall at the same rate.⁷ In this analysis Galileo practically came to the conclusion that a falling body does not resist its fall.

A thorough analysis of Galileo's discovery could have revealed an interesting similarity between that discovery and inertial motion. According to Newton's first law of motion (and Galileo's own experiments which had led him to the idea of inertial motion) different bodies move with the *same velocity by inertia* no matter whether they are heavy or light. So if heavy and light bodies fall with the same acceleration it is tempting to say that they move by inertia and because of this it does not matter whether they are heavy or light. However, the problem is obvious – how could they move by inertia if they accelerate?

Galileo (through Salviati) virtually arrived at the conclusion that a falling body does not resist its fall (Dialogues concerning two sciences , p. 447):

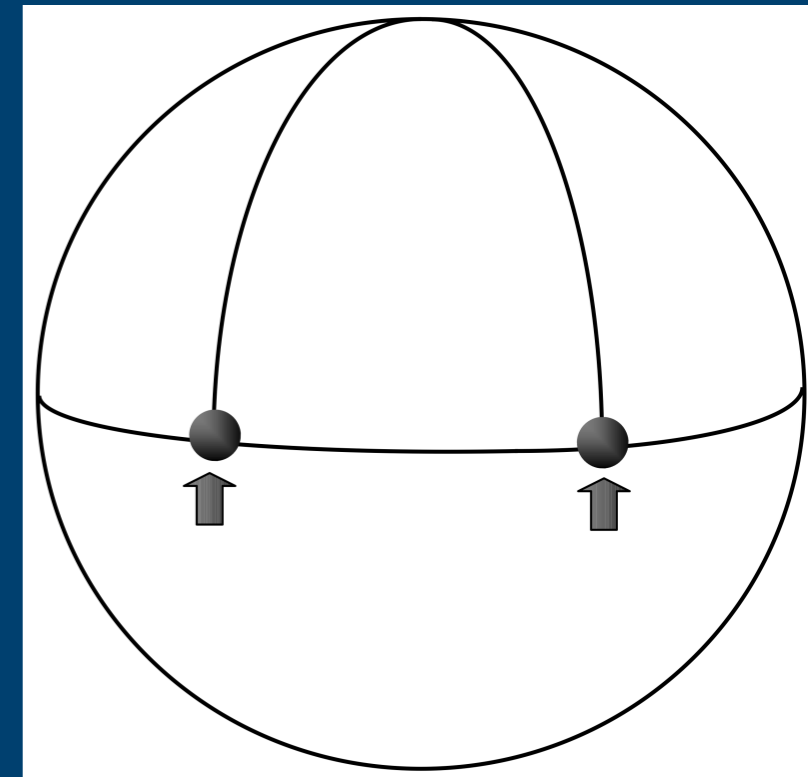
But if you tie the hemp to the stone and allow them to fall freely from some height, do you believe that the hemp will press down upon the stone and thus accelerate its motion or do you think the motion will be retarded by a partial upward pressure? One always feels the pressure upon his shoulders when he prevents the motion of a load resting upon him; but if one descends just as rapidly as the load would fall how can it gravitate or press upon him? Do you not see that this would be the same as trying to strike a man with a lance when he is running away from you with a speed which is equal to, or even greater, than that with which you are following him? You must therefore conclude that, during free and natural fall, the small stone does not press upon the larger and consequently does not increase its weight as it does when at rest.



If a falling body does not resist its fall, then the path to the idea that gravitational phenomena are manifestations of the curvature of spacetime would have been open to Minkowski:

The experimental fact that a falling particle accelerates (which means that **its worldtube is curved**), but offers no resistance to its acceleration (which means that its worldtube is not deformed) can be explained only if the worldtube of a falling particle is **both curved and not deformed**, which is impossible in the flat Minkowski spacetime where a curved worldtube is always deformed.

Such a worldtube can exist only in a non-Euclidean spacetime whose geodesics are naturally curved due to the spacetime curvature, but are not deformed.



Second scenario

Imagine that after his lecture *Space and Time* Minkowski found a very challenging mathematical problem and did not compete with Einstein for the creation of the modern theory of gravitation. But when Einstein linked gravitation with the geometry of spacetime Minkowski regretted his change of research interests and started to study intensely general relativity and its implications.

As a mathematician he would be appalled by what he saw as confusing of physics and geometry:

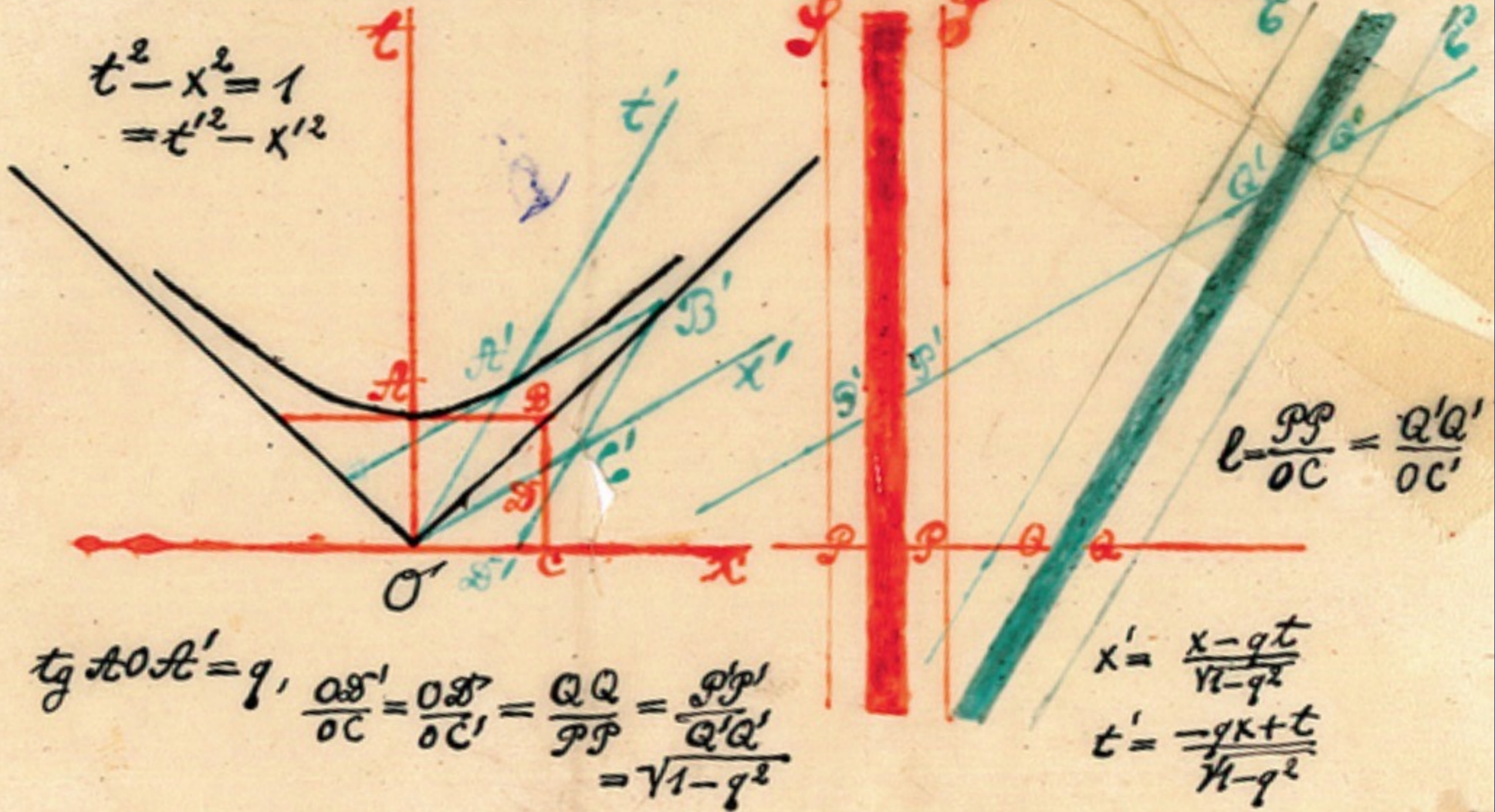
- The new theory of gravitation demonstrates that gravitational physics is in fact geometry of curved spacetime; no general relativity of anything can be found there.
- How could physicists say that in the framework of general relativity itself gravitational phenomena are caused by gravitational interaction? According to what general relativity **itself** tells us, gravity is not a physical interaction since by the geodesic hypothesis particles falling towards a planet and planets orbiting the Sun all move by inertia and inertia by its very nature presupposes no interaction.

- How could physicists talk about gravitational energy in the framework of general relativity? There is no gravitational field and no gravitational force; the gravitational field is at best a geometric not a physical field, and as such it does not possess any energy. In other words, there is no gravitational energy since energy is defined as the work done by gravitational forces, but there are no such forces in general relativity.
- Moreover, the mathematical formalism of general relativity itself refuses to yield a proper (tensorial) expression for gravitational energy and momentum.

Kontraktion der Elektronen

$$t^2 - x^2 = 1$$

$$= t'^2 - x'^2$$

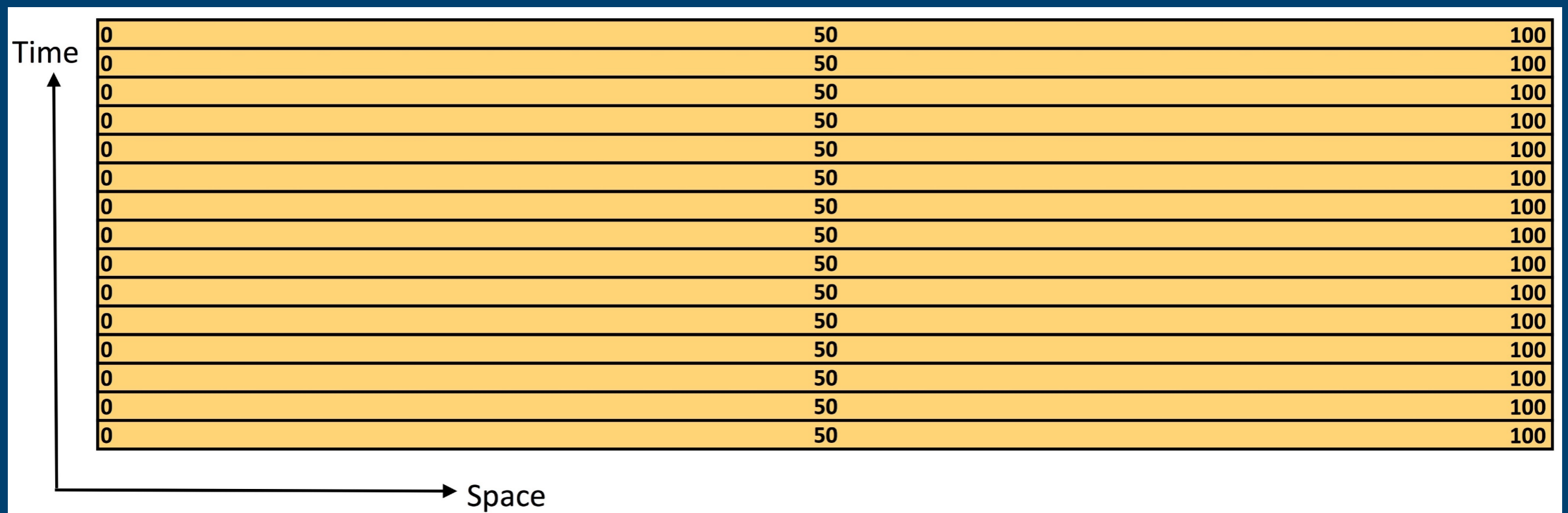


$$l = \frac{PQ}{OC} = \frac{Q'Q'}{OC'}$$

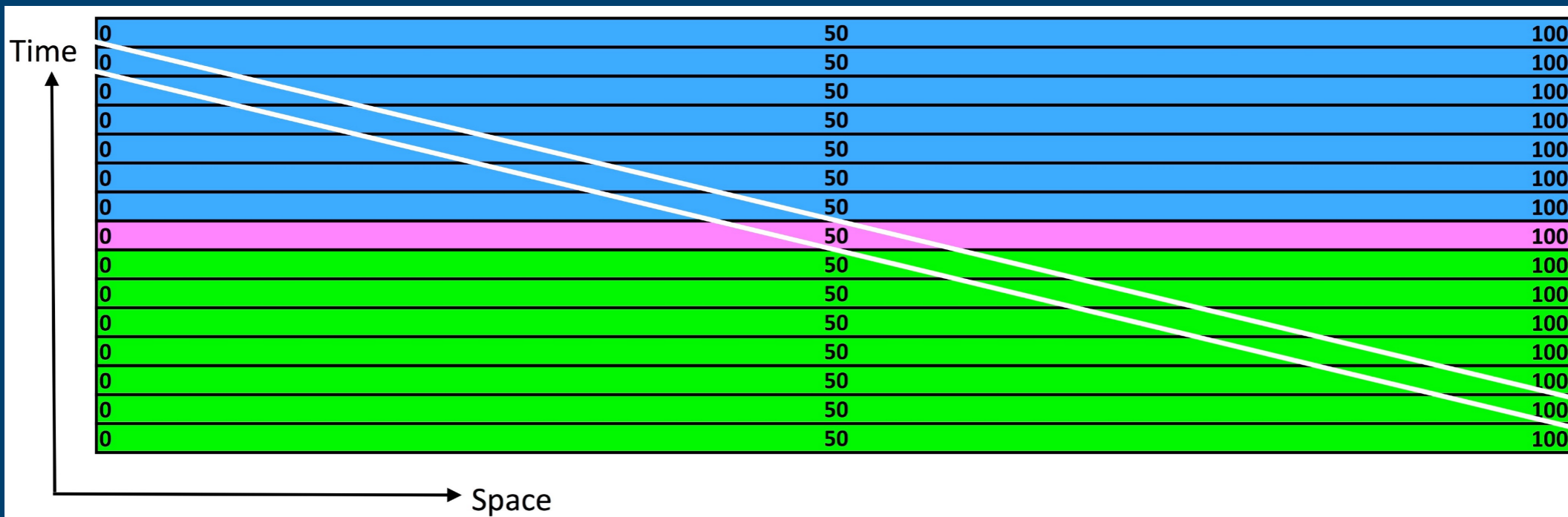
$$\text{tg } \angle AOA' = q, \quad \frac{OC}{OC'} = \frac{OB}{OB'} = \frac{PQ}{P'Q'} = \frac{Q'Q'}{Q'Q} = \frac{1}{\sqrt{1-q^2}}$$

$$x' = \frac{x - qt}{\sqrt{1-q^2}}$$

$$t' = \frac{-qx + t}{\sqrt{1-q^2}}$$







If Einstein had examined Minkowski's idea thoroughly he would have most probably considered and carefully analyzed the heretical possibility that **gravity is not a physical interaction**. Had he lived longer, Minkowski himself would have almost certainly arrived at this radical possibility. In 1921 Eddington even mentioned it explicitly:

Gravitation as a separate agency becomes unnecessary.

A. S. Eddington, The Relativity of Time, *Nature* **106**, 802-804 (17 February 1921); reprinted in: A. S. Eddington, *The Theory of Relativity and its Influence on Scientific Thought: Selected Works on the Implications of Relativity* (Minkowski Institute Press, Montreal 2015).