Minkowski's Program of Geometrizing Physics and General Relativity

Vesselin Petkov

Minkowski Institute
Montreal, Canada

MINKOWSKI INSTITUTE

OUTLINE

I will discuss three main issues:

- Hermann Minkowski initiated the first program of geometrizing physics and started to implement it.

- Contrary to common belief Einstein himself did not believe that general relativity geometrized physics.

- Strictly following Minkowski's original idea of geometrizing physics appears to suggest that gravitational phenomena are fully described in general relativity as manifestations of the non-Euclidean geometry of spacetime without the need to introduce the notion of gravitational interaction.

Disclaimer or rather clarification

Instead of giving a talk, for example, on calculating the anisotropic volume element in a non-inertial reference frame, which corrects both

- the erroneous factor of 1/2 in Fermi's potential and electric field of an electron supported in a gravitational field (1921) and
- the famous 4/3 factor in the self-force acting on the classical electron (which is still worth doing),

I prefer to talk about something that may look heretical to some. The reason is that one of the ways to deal with the unsuccessful attempts to create a theory of quantum gravity is to examine rigorously the taken-for-granted assumption that gravity is a physical interaction (I am not sure that quantizing spacetime means quantizing gravity).

I am pretty convinced (based on rereading his papers) that Minkowski would have examined this heretical possibility, had he lived longer.

To my knowledge, only Eddington entertained the thought that gravitation might not be a physical interaction:

- "Gravitation as a separate agency becomes unnecessary" [1]
- "An electromagnetic field is a "thing;" gravitational field is not, Einstein's theory having shown that it is nothing more than the manifestation of the metric" [2]

[1] A.S. Eddington, The Relativity of Time, *Nature* 106, 802-804 (17 February 1921); reprinted in: A. S. Eddington, *The Theory of Relativity and its Influence on Scientific Thought: Selected Works on the Implications of Relativity* (Minkowski Institute Press, Montreal 2015) pp. 27-30, p. 30

[2] A.S. Eddington, *The Mathematical Theory of Relativity* (New publication by Minkowski Institute Press, Montreal 2016) p. 233.

Minkowski decoded the hidden message in all failed experiments to detect absolute motion - from Galileo to MM experiment - captured in the relativity postulate:

The profound physical reason of why all those experiments failed is that the world is four-dimensional!

To explain the null result of the MM experiment, Lorentz formally introduced a local time for an observer on Earth that is different from the true time measured by an observer at absolute rest. Einstein postulated that the two times should be regarded on equal footing. Minkowski noticed that different times necessarily imply different spaces (= different classes of simultaneous events), which makes it exceedingly clear (to a mathematician) that *different times and different spaces are only possible in a four-dimensional world*:

MINKOWSKI INSTITUTE

“Hereafter we would then have in the world no more ***the*** space, but an infinite number of spaces analogously as there is an infinite number of planes in three-dimensional space. Three-dimensional geometry becomes a chapter in four-dimensional physics.”

As many spaces are impossible in a three-dimensional world, it is clear why Minkowski regarded reality as a four-dimensional world. Unfortunately there are physicists who think either that

- spacetime is nothing more than a mathematical continuum (e.g., D.N. Mermin) [misconception with far reaching negative implications]
- the question of the reality of spacetime belongs to philosophy [physics at its worst].

If the real world were three-dimensional, there would exist a **single** space, i.e. a **single** class of simultaneous events (a **single** time), which would mean that simultaneity and time would be absolute in contradiction with both the theory of relativity and, most importantly, the experiments which confirmed its predictions (e.g., it is an experimental fact, used every second by the GPS, that observers in relative motion have different times, which is impossible in a three-dimensional world).

MINKOWSKI INSTITUTE

Minkowski had clearly realized that four-dimensional physics was in fact spacetime geometry since all particles which appear to move in space and last in time are in reality a forever given web of the particles' worldlines in spacetime. Then Minkowski outlined his program of geometrization of physics:

“The whole world presents itself as resolved into such worldlines, and I want to say in advance, that in my understanding the laws of physics can find their most complete expression as interrelations between these worldlines.”

Then Minkowski reported the first instances of the implementation of his program – by regarding the discovered by him four-dimensional physics as spacetime geometry (or in Minkowski's terminology "world-geometry") he explained

- why inertial observers in relative motion have different times (that was merely postulated by Einstein) and different spaces (first realized by Minkowski), which is impossible in a three-dimensional world (it is this explanation that made Minkowski realize that the true reality is a four-dimensional world – die Welt or spacetime);

- the absolute distinction between inertial (non-resistant) and accelerated (resistant) motion since they are represented by distinct timelike worldlines (or rather worldtubes) – straight and curved (deformed), respectively; this geometrical distinction immediately leads to an attractive explanation of the corresponding physical distinction between inertial and accelerated motion (because Minkowski regarded the particles' worldtubes as real) – inertial motion cannot be detected experimentally, whereas accelerated motion can be (because an accelerated particle is a deformed worldtube in spacetime, which resists its static deformation);

- the equivalence of all inertial observers (or inertial frames), that was also postulated by Einstein – they are equivalent because they are all straight worldtubes; this explanation also explains the physical meaning of the relativity principle – physical laws are the same for all inertial observers because each observer describes the physical phenomena exactly in the same way – in his own space, in which he is at rest, and by using his own time;

- the postulated by Einstein constancy of the speed of light: it is the same for all observers, because each of them measures it exactly in the same way – in his own space, in which he is at rest, and by using his own time;

- the physical meaning of length contraction – the relativistic length contraction of a rod, for example, is possible if and only if the rod's worldtube is a real four-dimensional object, because only then two inertial observers' spaces (which form an angle corresponding to the observers' relative velocity) can intersect the rod's worldtube at two cross-sections (measured as two three-dimensional rods) of different length.

However, most physicists seem to believe that it was Einstein's general relativity which first geometrized physical phenomena. This is an unfortunate historical injustice on two counts:

- many relativists are probably not fully aware that it was Minkowski who first introduced the program to geometrize all physics (not just gravitation) – to regard the four-dimentional physics as spacetime (or World) geometry – and who started to employ this program to the physics of flat spacetime;

- contrary to common belief, Einstein himself did not believe that general relativity geometrized gravitation: "I do not agree with the idea that the general theory of relativity is geometrizing Physics or the gravitational field" [1]. Einstein looked at the mathematical formalism of general relativity as pure mathematics and regarded gravitation as a physical interaction involving exchange of gravitational energy and momentum.

[1] A letter from Einstein to Lincoln Barnett from June 19, 1948; quoted in D. Lehmkuhl, Why Einstein did not believe that General Relativity geometrizes gravity. *Studies in History and Philosophy of Physics*, Volume 46, May 2014, pp. 316-326.

Despite that according to the currently accepted understanding of general relativity, which was initiated and greatly influenced by Einstein himself, gravitation is a physical interaction involving exchange of gravitational energy and momentum, I think, following Eddington, that taken at face value general relativity geometrizes gravitation **fully**.

Had he lived longer, Minkowski would have certainly seen general relativity as a triumph of his program of geometrizing physics and would have very likely demonstrated that the only rigorous interpretation of the mathematical formalism of general relativity is that gravitation is nothing more than manifestation of the non-Euclidean geometry of spacetime in full agreement with his program.

Indeed, as a mathematician (who would not allow anything external, like gravitational energy and momentum, to be smuggled into the theory) Minkowski might have concluded that all gravitational phenomena are completely explained in general relativity as manifestations of the non-Euclidean geometry of spacetime without the need to assume that gravitational interaction is causing the gravitational phenomena:

- a particle, whose worldline is geodesic, is a free particle moving by inertia (the geodesic principle); therefore the motion of bodies falling toward the Earth's surface and of planets orbiting the Sun (whose worldtubes are geodesic) is inertial, i.e., interaction-free, because the very essence of inertial motion is motion which does not involve any interaction (and any exchange of energy and momentum) whatsoever.

- the deformed worldtube of a particle on the Earth's surface may be regarded as giving rise to an inertial force (a static restoring force in the particle's deformed worldtube), which has been traditionally called gravitational force or the particle's weight.

- what is called gravitational energy appears to be inertial energy arising from the work done by inertial forces with which free particles (moving by inertia), represented by geodesic worldtubes, resist any deformation of their worldtubes (i.e., resist when prevented from moving by inertia); for this reason gravitational waves could not carry gravitational energy as Eddington stressed it in his fundamental treatise on the mathematical and physical foundations of general relativity *The Mathematical Theory of Relativity:*

"The gravitational waves constitute a genuine disturbance of space-time, but their energy, represented by the pseudo-tensor tνμ, is regarded as an analytical fiction" [1, p. 260] (it cannot be regarded as an energy of any kind for the well-known reason that "It is not a tensor-density and it can be made to vanish at any point by suitably choosing the coordinates; we do not associate it with any absolute feature of world-structure," ibid, p. 136).

- A closer look at Feynman's famous sticky bead argument (regarded by many as demonstrating that gravitational waves do carry gravitational energy) reveals that kinetic, not gravitational, energy is converted into heat; more precisely inertial not kinetic energy is converted into heat because the bead is initially at rest and does not possess kinetic energy (a simple calculation shows that kinetic and inertial energy are the same thing, but the adequate term, as the sticky bead argument demonstrated, is inertial energy since it reveals the very origin of this energy – the work done by inertial forces when particles are prevented from moving by inertia).

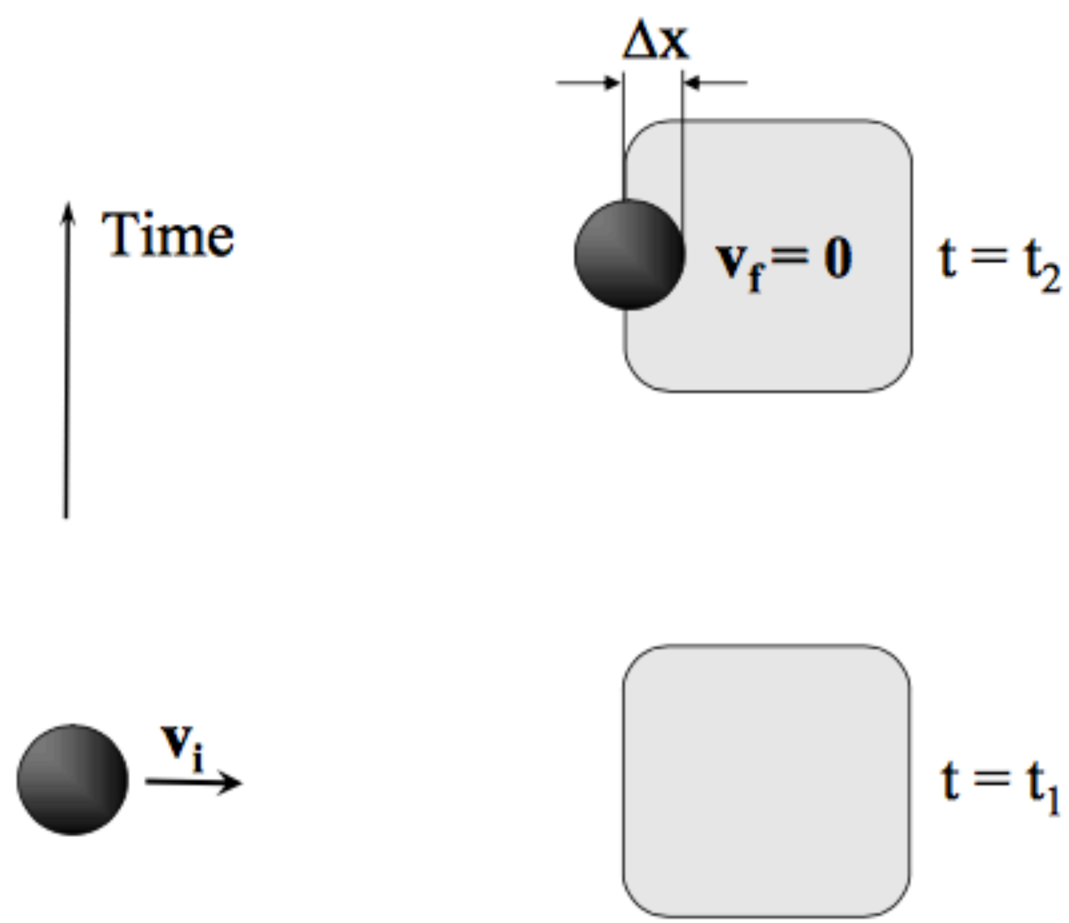


Figure 6.3: A massive plastic block is deformed when hit by a ball moving by inertia. Traditionally, it is stated that the ball's kinetic energy converts into a deformation energy. However, a deep physical explanation reveals that the ball's energy is inertial energy since the deformation is caused by the work done by the real inertial force with which the ball resists its deceleration

“College” proof that kinetic energy is, in fact, inertial energy

Using the relation between v_i, v_f, a and the distance Δx in the case of deceleration

$$v_f^2 = v_i^2 - 2a\Delta x$$

and taking into account that $v_f = 0$ we find

$$a = \frac{v_i^2}{2\Delta x}.$$

Then for the ball's inertial energy E_i we have

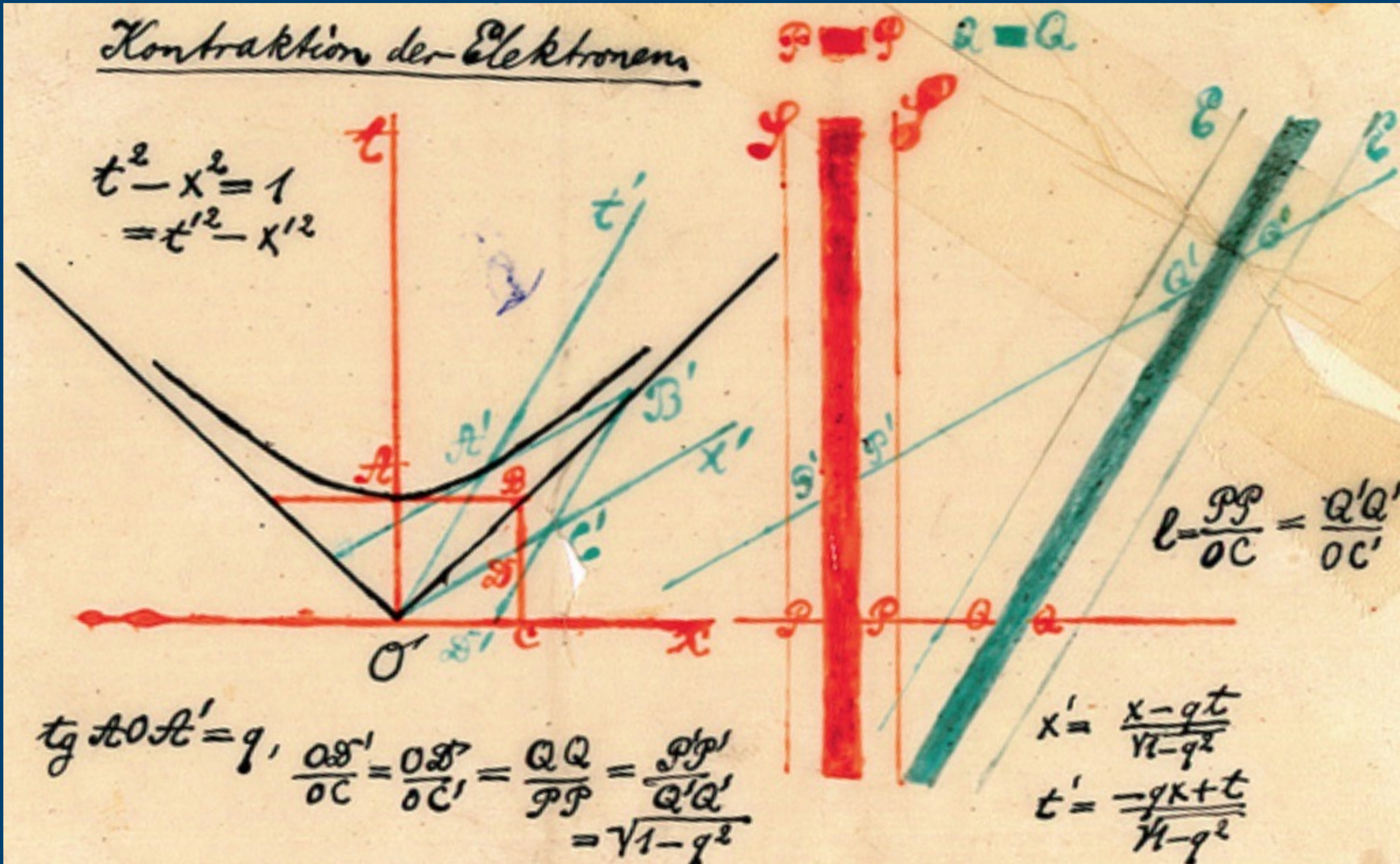
$$E_i = W = F\Delta x = ma\Delta x = \frac{1}{2}mv_i^2.$$

Conclusion

- In his 1908 lecture Space and Time, Hermann Minkowski initiated the first program of geometrizing physics and started to implement it.

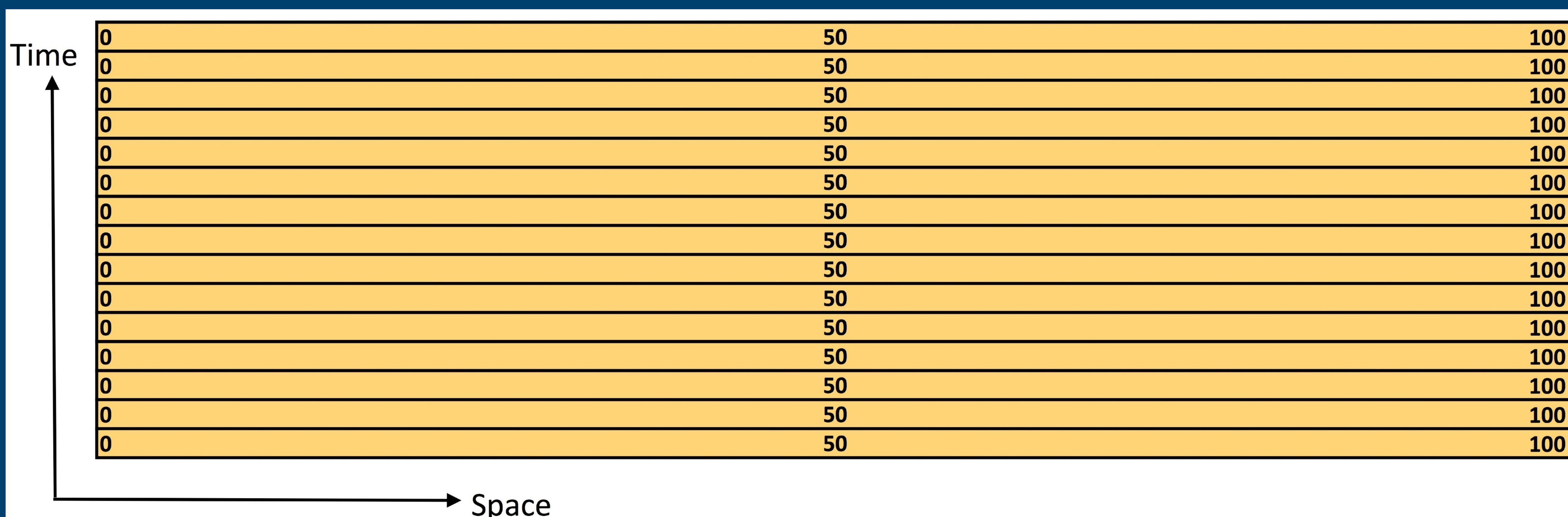
- Contrary to common belief Einstein himself did not believe that general relativity geometrized physics.

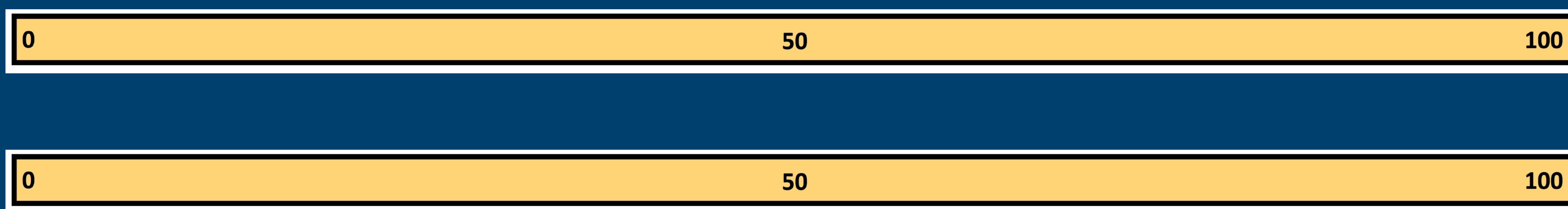
- Minkowski's original idea of geometrizing physics might have led him to the conclusion that gravitational phenomena are fully described in general relativity as manifestations of the non-Euclidean geometry of spacetime without the need to introduce the notion of gravitational interaction.

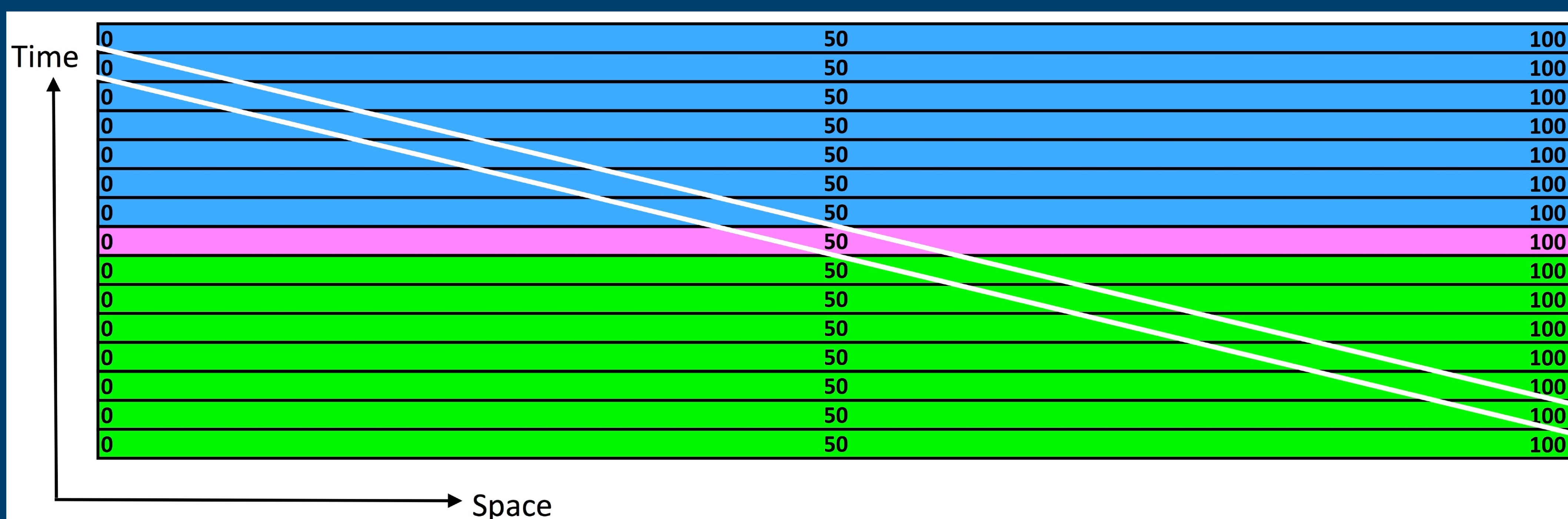


MINKOWSKI INSTITUTE


0
50
100


Time
0 50 100
0 50 100
0 50 100
0 50 100
0 50 100
0 50 100
0 50 100
0 50 100
0 50 100
0 50 100
0 50 100
0 50 100
0 50 100
0 50 100
0 50 100
Space


0
50
100
0
50
100



MINKOWSKI INSTITUTE