

Appendix B: On inertial forces, inertial energy and the origin of inertia

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Abstract

As a result of the open question of inertia the status of inertial forces has been a difficult subject in physics with implications for the proper understanding of the force of weight in general relativity where gravity is not a force, but a manifestation of the spacetime curvature. The purpose of this paper is fourfold. First, to state explicitly when the inertial forces are fictitious and when real. Second, to provide a virtually self-evident derivation, which demonstrates that kinetic energy is in fact inertial energy – the energy equal to the work done by inertial forces. Third, to stress that weight, which has been traditionally regarded as a gravitational force, is an inertial

force in general relativity. Fourth, to outline what relativity implies about the origin of inertia.

Keywords Inertial forces, inertial energy, kinetic energy, geodesic hypothesis, origin of inertia, four-dimensional stress

The best way to approach the issue of inertial forces is by recalling the definition of mass which has been adopted since Newton – mass is the measure of the *resistance* a particle offers to its acceleration. It is this resistance, commonly called inertia, which *experimentally* distinguishes accelerated from inertial motion. Due to the fact that the presence or the absence of a particle's resistance to its motion is *absolute* or frame-independent, both accelerated and inertial motion are absolute or frame independent. An accelerating particle's resistance allows its state of absolute acceleration to be detected in any reference frame. Similarly, the absence of resistance to a particle's motion makes the detection of the particle's inertial motion possible in any reference frame. For this reason a particle's resistance to its motion is a necessary and sufficient condition for it to be in a state of absolute acceleration caused by some interaction. Conversely, if a particle does not resist its motion, it is a free particle, which is not subject to any interactions and moves by inertia.

In his spacetime formulation of special relativity Minkowski provided rigorous criteria for inertial and accelerated motion [1] – a free particle, which moves by inertia, is a straight timelike worldline in Minkowski spacetime, whereas the timelike worldline of an accelerating particle is curved. These criteria show that in spacetime the absoluteness of accelerated and inertial motion become even more understandable – the straightness of a timelike worldline (representing inertial motion) and the curvature or rather the *deformation* of a timelike worldline (representing accelerated motion) are absolute (frame independent)

properties of worldlines. Therefore it is the *deformation* of the worldline of an accelerating particle that makes the particle's acceleration absolute¹⁵. In such a way Minkowski's spacetime representation of special relativity unequivocally supported Newton's view of absolute acceleration and disproved Mach's arguments that acceleration, like velocity, is also relative¹⁶. Also, the proper relativistic understanding of the absoluteness of acceleration demonstrates that absolute acceleration merely reflects the deformation of an accelerating particle's worldline and does not imply some absolute space with respect to which the particle accelerates.

Now the distinction between fictitious and real inertial forces can be demonstrated by a simple example involving an accelerating elevator. Let two elevators I and N be at relative rest far away from gravitating masses and let a ball be floating in the middle of N . At a given moment N starts to accelerate translationally and observers inside it see that the ball starts to fall (accelerate) towards the elevator's floor. The apparent accelerated motion of the ball can be formally regarded as caused by a force. However, due to the fact that the presence of acceleration is absolute (frame independent), observers in both the inertial elevator I and the non-inertial elevator N agree that it is N that accelerates, not the ball; it is N 's floor that *in reality*

¹⁵ Acceleration as a deformation of a geodesic worldline is absolute in both special and general relativity. There is a second acceleration in general relativity which, however, is not related to a deformation of the geodesic worldline of an apparently accelerating particle and it is *non-resistant* since it is relative or apparent – the absence of parallel, or rather congruent, geodesic worldlines in non-Euclidean spacetime leads to geodesic deviation, which manifests itself as a relative acceleration in general relativity.

¹⁶ Mach argued that one could not say whether or not a *single* particle in the Universe accelerates. By contrast, that situation in spacetime is crystal clear – the worldline of a single particle in the Universe is either straight or deformed, which means that the particle is either moving by inertia or with an acceleration.

approaches the ball. And there is no relativity here since an accelerometer attached to the ball detects no acceleration. This is best seen in the spacetime diagram depicted in Fig. 6.2. For this reason the force formally introduced to explain the ball's apparent acceleration in N is not a real force; it is a fictitious inertial force, which can be imagined when the inertial motion of the ball is described in the non-inertial elevator N . The situation changes when N 's floor reaches the ball and starts to accelerate it. The ball *resists* the change in its inertial state and exerts a real translational inertial force back on the floor.

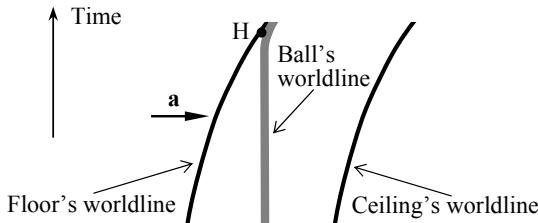


Figure 6.2: A starting to accelerate (with acceleration \mathbf{a}) elevator N is represented by the worldlines of its ceiling and floor. A ball which was floating in the middle of N before N started to accelerate is represented by its worldline. The ball's worldline is straight, which means that it does not accelerate and therefore moves by inertia. It is the floor's worldline that converges towards the straight worldline of the ball, or in the ordinary three-dimensional language it is the floor that *in reality* approaches the ball. At event H the floor hits the ball and starts to accelerate it, the ball's worldline is deformed and the ball resists its acceleration by acting back on the floor with a *real* inertial force.

This distinction between fictitious and real inertial forces in the case of translational acceleration holds also for the cases of rotational non-inertial motion. Translational, centrifugal, and

Coriolis inertial forces are fictitious when a *free* particle (moving non-resistantly by inertia) *only appears to be accelerated by a fictitious inertial force* when described in a non-inertial reference frame. When the particle is compelled to move with the non-inertial frame's acceleration, it starts to resist the change in its inertial motion and exerts a *real inertial force* on the mover that accelerates it.

What also might contribute to a better understanding of the status of inertial forces is the fact that *real inertial forces do work*, which implies that kinetic energy is rather *inertial energy*. In the above case the deformation on N 's floor (resulting from the collision of the ball and the floor) is caused by the real inertial force with which the ball resists its acceleration. Therefore the work done by the ball's inertial force, which is equal to its inertial energy, converts into a deformation energy. So far inertial energy has been called kinetic energy. But such a name does not reveal the true nature of the ball's energy responsible for the deformation on the N 's floor – the ball's inertia, i.e. its resistance to the change in its inertial state.

The qualitative argument that kinetic energy is actually inertial energy has a straightforward quantitative counterpart. That inertial energy – the work done by inertial forces – is equal to kinetic energy is easily demonstrated by an example depicted in Fig. 6.3. At moment $t = t_1$ a ball travels at constant “initial” velocity v_i towards a huge block of some plastic material; we can imagine that the block is mounted on the steep slope of a mountain. Immediately after that the ball hits the block, deforms it and is decelerated. At moment $t = t_2$ the block stops the ball, that is, the ball's final velocity at t_2 is $v_f = 0$ (the block's mass is effectively equal to the Earth's mass, which ensures that $v_f = 0$). According to the standard explanation it is the ball's kinetic energy $E_k = (1/2)mv_i^2$ which transforms into a deformation energy. But a proper physical explanation demon-

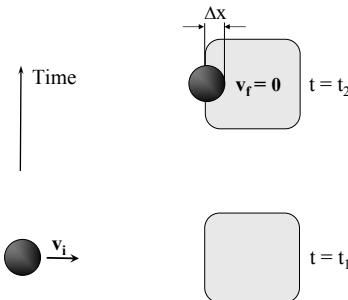


Figure 6.3: A massive plastic block is deformed when hit by a ball moving by inertia. Traditionally, it is stated that the ball's kinetic energy converts into a deformation energy. However, a deep physical explanation reveals that the ball's energy is inertial energy since the deformation is caused by the work done by the real inertial force with which the ball resists its deceleration

strates that the energy of the ball, which is transformed into deformation energy, is its inertial energy E_i , because the ball resists its deceleration a and it is the work $W = F\Delta x$ (equal to E_i) done by the *inertial force* $F = ma$ that is responsible for the deformation of the plastic material.

Using the relation between v_i , v_f , a and the distance Δx in the case of deceleration

$$v_f^2 = v_i^2 - 2a\Delta x$$

and taking into account that $v_f = 0$ we find

$$a = \frac{v_i^2}{2\Delta x}.$$

Then for the ball's inertial energy E_i we have

$$E_i = W = F\Delta x = ma\Delta x = \frac{1}{2}mv_i^2.$$

Therefore the inertial energy of the ball is indeed equal to what has been descriptively (lacking physical depth) called kinetic energy.

Minkowski's distinction between inertial and non-inertial motion has been generalized in general relativity by the *geodesic hypothesis* – a free particle is a timelike *geodesic* worldline in curved spacetime. The geodesic hypothesis is “a natural generalization of Newton's first law” [2], that is, “a mere extension of Galileo's law of inertia to curved spacetime” [3]. Therefore *in general relativity a particle, whose worldline is geodesic, moves by inertia*. A particle, whose worldline is deformed (that is, is not a geodesic), resists its deviation from its geodesic (i.e. inertial) path in spacetime, and exerts a real inertial force on the obstacle that deforms the particle's worldline.

The geodesic hypothesis has been proved by the experimental fact that particles falling towards the Earth's surface *do not resist* their fall¹⁷ (a falling accelerometer reads zero acceleration or rather zero resistance), which means that they indeed move by inertia while falling. If a falling particle were subject to a gravitational force it would *resist* its fall (its apparent acceleration) because by Newton's second law *a force is only needed to overcome an accelerating particle's resistance*. When a falling particle hits the ground it is prevented from moving by inertia and resists its resulting absolute acceleration¹⁸ (while being at rest on the ground, the particle's worldline is *deformed* and

¹⁷In the case of small particles tidal effects can be safely ignored. But even if tidal effects are taken into account, the tidal friction has nothing to do with the resistance a particle which is subject to a force offers to its acceleration. The tidal effects are merely a manifestation of the spacetime curvature.

¹⁸It follows from here that, like the inertial mass, the passive gravitational mass can be also defined as the measure of the resistance a particle offers to its acceleration, which sheds additional light on the equivalence of the two masses [4].

by the generalized Minkowski criterion the particle is subject to absolute acceleration). Two things are now evident – (i) the particle's weight is the resistance force, which it exerts on the ground, and (ii) that resistance force is *inertial* since it is the force with which the particle *resists* the change in its inertial motion (its fall).

The relativistic explanation of the absoluteness of acceleration as a *deformation* of the worldline or rather the worldtube of an accelerating particle provides an unanticipated insight into the origin of inertia. The resistance an accelerating particle offers to its acceleration (i.e. to the deformation of its worldtube) can be regarded as a manifestation of a static restoring force caused by a four-dimensional stress that arises in the deformed worldtube¹⁹ of the accelerating particle (like the restoring force arising when an ordinary three-dimensional rod is deformed)

¹⁹This explanation of the origin of inertia presupposes that the worldtubes of particles are real four-dimensional objects, which is a direct consequence of Minkowski's view of special relativity as a theory of an absolute four-dimensional world and particularly of his explanation of length contraction. In Minkowski's explanation of this relativistic effect the instantaneous three-dimensional spaces of two observers in relative motion intersect the worldtube (Minkowski called it the world strip) of a body at different angles and the two resulting three-dimensional cross-sections have different lengths. It is obvious that these cross-sections of the worldtube of the *same* three-dimensional body would be impossible if its worldtube were not real, i.e. if it were a mere geometrical abstraction [1] (see also [4, Chap. 5], [5]). In fact, the reality of the worldtubes of physical objects and the four-dimensionality of the world itself (at least at the macro scale) follows from the experimental evidence supporting the relativity principle as first realized by Minkowski. He noticed that as observers in relative motion, according to the relativity principle, have *different times* (the realization of which led Einstein to the special theory of relativity), it follows that the observers also have *different spaces* – “Hereafter we would then have in the world no more *the space*, but an infinite number of spaces analogously as there is an infinite number of planes in three-dimensional space. Three-dimensional geometry becomes a chapter in four-dimensional physics.” [1]. Obviously, many three-dimensional spaces imply a four-dimensional world.

[4, 5]. Therefore relativity implies that inertia has a *local* origin – it is the accelerating particle *itself* that resists the deformation of its worldtube. This means that inertia is not a non-local phenomenon that is caused by the distant masses as Mach argued. It might be tempting to say that what determines the shape of a free particle’s geodesic worldtube (which when deformed resists its deformation) are all the masses in the Universe in line with Mach’s view of inertia as caused by the distant masses. However, such a temptation would be misleading since in curved space-time it is the nearby mass that is essentially responsible for the shape of the geodesics in its vicinity. The shape of the geodesic worldline of a particle falling towards the Earth, for example, is predominantly determined by the Earth’s mass and the distant masses have practically zero contribution.

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