but coordinate time (the time between the same readings of the clock measured in the inertial frames moving relative to the clock) is greater; for a more detailed discussion of relativistic mass see [33, pp. 114-116].

It is evident that the spacetime explanation of the origin of inertia makes sense only if the worldtube of an accelerating particle is a *real* four-dimensional object. This explanation of inertia shows that even for practical reasons the implications of the reality of spacetime should be thoroughly explored because, for example, if we understand the origin of inertia we will be in a position to determine whether inertia can be controlled. But, obviously, the issue of the reality of spacetime (i.e. that the world is four-dimensional) should be resolved first.

We already saw that the experimental evidence behind the relativity principle could not be explained if the world were three-dimensional, i.e., if the absolute four-dimensional world (spacetime) were just a mathematical abstraction. This has been realized not only by Minkowski, but also by the majority of physicists who specialized in spacetime physics as seen from what some of them wrote (quoted at the end of Chapter 1). Here I will only quote again Eddington's direct answer to the question of the reality of spacetime since it was given in 1921 not long after Minkowski's discovery (Eddington calls the spaces and times of observers in relative motion *fictitious* since such spaces do not represent anything real because they are *imaginary* three-dimensional cross-sections of spacetime, exactly like the xy planes of different coordinate systems are imaginary two-dimensional cross-sections of space; analogously, the observers' times are fictitious because they can be chosen along the worldline of *any* uniformly moving particle, exactly like the z directions in space can be freely chosen[5]) [50, p.803]:

> It was shown by Minkowski that all these fictitious spaces and times can be united in a single continuum of four dimensions. The question is often raised whether this four-dimensional space-time is real, or merely a mathematical construction; perhaps it is sufficient to reply that it can at any rate not be less real than the fictitious space and time which it supplants.

Let us now examine what seems to be the most spectacular proof of a world view in the history of science. We owe both the world view

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[5] This comparison can be made rigorous if it is noted that the z directions in space can be chosen in any direction without any restriction, whereas the time axes in spacetime can be chosen only along timelike worldlines representing particles. In the same way the xy planes in space can be freely chosen without restrictions, whereas the spaces of observers in spacetime should be along spacelike directions.

(the spacetime world view) and the proof to Minkowski – the experiments which confirmed the kinematical relativistic effects would be impossible if spacetime (i.e. the four-dimensional world) did not exist; stated another way, the relativistic experiments would be impossible in a three-dimensional world.

Let us start with relativity of simultaneity or Minkowski's version of this consequence of special relativity – observers in relative motion have different spaces. As discussed in Chapter 1, a space constitutes a class of simultaneous events and therefore, having different spaces, observers in relative motion have different classes of simultaneous events (relativity of simultaneity). If the world were three-dimensional, space would be absolute since there would exist only one space that would be shared by all observers in relative motion. As space is a single class of simultaneous events, absolute space implies absolute simultaneity and therefore absolute time as well. All this is in a clear contradiction with special relativity.

Two things about relativity of simultaneity were mentioned in Chapter 1. First, the proof in the above paragraph, that no relativity of simultaneity is possible in a three-dimensional world, is valid only if existence is absolute. Although Minkowski's arguments clearly demonstrated that no relative quantities would be possible if an underlying absolute reality did not exist, we will see below that the idea to relativize existence (to preserve the three-dimensionality of the world) contradicts the experiments that confirmed the twin paradox effect. Second, relativity of simultaneity has never been directly tested experimentally, but length contraction and time dilation, which are specific manifestations of relativity of simultaneity, have been experimentally confirmed. Therefore, relativity of simultaneity, taken even alone, is sufficient to prove the reality of spacetime.

A more detailed argument is Minkowski's explanation of the deep physical meaning of length contraction of two bodies in relative motion. The essence of his explanation is that length contraction is a manifestation of the reality of the bodies' worldtubes (Minkowski called them strips). This can be best understood from Fig. 1 of Minkowski's paper "Space and Time" (the right-hand part of which is reproduced in Fig. 5.5 here) – length contraction would be *impossible* if the worldtubes of the two bodies, represented by the vertical and the inclined strips in Fig. 5.5, did not exist and were nothing more than abstract geometric constructions. To see this even more clearly consider only the body represented by the vertical worldtube. The three-dimensional cross-section PP, resulting from the intersection of the body's worldtube and the space of an observer at rest with respect to the body, is the body's proper length. The three-dimensional cross-section $P'P'$, resulting from the intersection of the body's worldtube and the space of an observer moving with respect to the body, is the relativistically contracted length of the body measured by that observer[6]. Minkowski stressed that "This is the meaning of the Lorentzian hypothesis of the contraction of electrons in motion" [12, p.116].

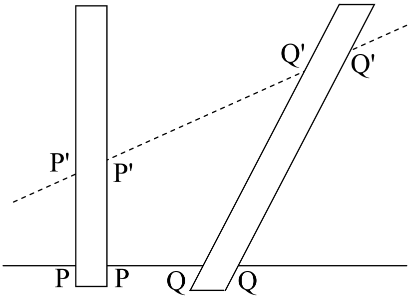

Figure 5.5: The right-hand part of Minkowski's Fig. 1

By demonstrating that the length contraction of a body is a manifestation of the reality the body's worldtube (and therefore of the reality of the absolute four-dimensional world) Minkowski also showed that this effect involves no deformation and no force causing the shortening of the body's length; as seen in Fig. 5.5 the contracted body measured by the observer moving relative to the body is simply *a different cross-section* $P'P'$ of the body's worldtube, which is shorter that the cross-section PP measured by the observer at rest with respect to the body. So, the length contraction effect is a nice illustration of the essence of Minkowski's four-dimensional physics – that *four-dimensional physics is spacetime geometry.*

It should be stressed that *the worldtube of the body must be real in order that length contraction be possible* because, while measuring the *same* body, the two observers in relative motion measure *two* three-dimensional bodies represented by the cross-sections PP and $P'P'$ in Fig. 5.5. This is not so surprising when one takes into account relativity of simultaneity and the fact that a spatially extended three-dimensional object is defined in terms of *simultaneity* – all parts of a body taken *simultaneously* at a given moment. If the worldtube of the body were an abstract geometric construction and what existed

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[6] The cross-section $P'P'$ only appears longer than PP because a fact of the pseudo-Euclidean geometry of spacetime is represented on the Euclidean surface of the page.

were a single three-dimensional body (a single class of simultaneous events) represented by the proper cross-section PP, both observers would measure the *same* three-dimensional body of the *same* length, i.e. the *same* class of simultaneous events, which means that simultaneity would be absolute.

Length contraction was tested experimentally, along with time dilation, by the muon experiment in the muon reference frame (see for instance [69]).

Length contraction of a body, also taken even alone, is sufficient to prove that this relativistic effect would be impossible if the body's worldtube were not real, that is, if the world were three-dimensional. Due to this fact, let us examine a thought experiment which visualizes Minkowski's explanation of length contraction.

Figure 5.6: An ordinary meter stick.

The thought experiment clearly demonstrates that length contraction of a meter stick would be impossible if the meter stick existed as a three-dimensional body (not a worldtube). An ordinary meter stick (Fig. 5.6) is at rest with respect to an observer A. What is shown in Fig. 5.6 is what we perceive and take for granted that it is what really exists. According to Minkowski, however, the meter stick exists equally at all moments of its history and what is ultimately real is the worldtube of the meter stick as shown in Fig. 5.7.

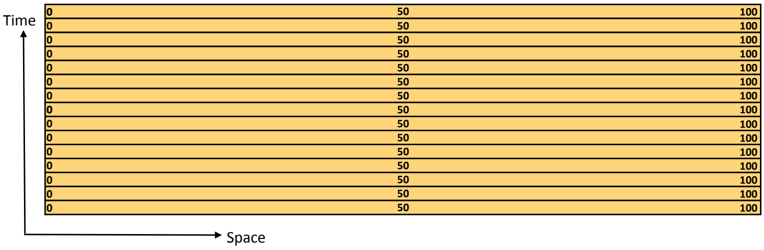

Figure 5.7: The worldtube of the meter stick.

Assume that another meter stick at rest in another observer's (observer B's) reference frame moves relative to the first one at a distance 1 mm above it. Let us assume that at the event M the middle point of B's meter stick is instantaneously above the middle point of A's meter stick. Lights are installed inside A's meter stick, which change their color *simultaneously* at every instant in A's frame. At the event of the meeting M all lights are red in A's frame. At all previous moments all lights were green. At all moments after the meeting all lights would be blue. When A and B meet at event M this event is present for both of them. At that moment all lights of A's meter stick will be *simultaneously red for* A. In other words, the present meter stick for A is red (that is, all parts of A's meter stick, which exist simultaneously for A, are red), All moments before M, when all lights of the meter were green, are past for A, whereas all moments when the meter stick will be blue are in A's future.

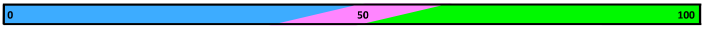

Figure 5.8: Relativistically contracted meter stick measured by observer B.

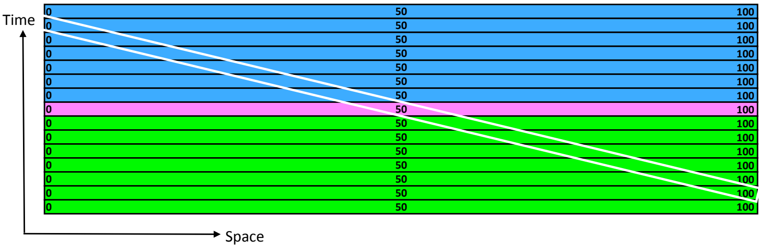

Figure 5.9: The worldtube of the meter stick with different colors.

Imagine that B's meter stick contains cameras, instead of lights, at every point along its length. At the event of the meeting M all cameras take snapshots of the parts of A's meter stick which the cameras face. All snapshots are taken simultaneously in B's reference frame. Even without looking at the pictures taken by the cameras it is clear that not all pictures will show a red part of A's meter stick, because what is simultaneous for A is not simultaneous for B. When the picture of A's meter stick is assembled from the pictures of all cameras it would show two things as depicted in Fig 5.8 – (i) A's meter stick photographed by B is shorter, and (ii) only the middle part of the picture of A's meter stick is red; half is green and the other half is blue. So what is past (green), present (red), and future (blue) for A exists *simultaneously* as present for B. But this is only possible if the meter stick is the worldtube as shown in Fig. 5.9. The instantaneous space of B corresponding to the event M intersects the worldtube of the meter stick at an angle and the resulting three-color "cross section" is what is measured by B – a different three-dimensional meter stick, which is shorter[7] than the meter stick measured by A.

It should be emphasized again that no length contraction would be possible if the meter stick's worldtube did not exist as a four-dimensional object. Otherwise, if the meter stick were a three-dimensional object, both observers would measure the *same* three-dimensional meter stick (the same set of *simultaneously* existing parts of the meter stick), which would mean that the observers would share the same (absolute) class of simultaneous events in a clear contradiction with relativity.

Now let us see that time dilation would also be impossible if the worldtubes of two digital clocks A and B in relative motion, shown in Fig. 5.10, did not exist and the clocks were the familiar three-dimensional bodies.

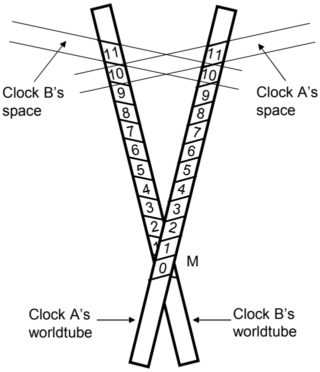

Figure 5.10: Reciprocal time dilation of two digital clocks

When the clocks meet at event M their readings are set to zero. Let two observers A and B be at rest with respect to clocks A and B, respectively. The two observers are performing identical experiments, which last ten seconds. The duration of the experiments is measured by the corresponding clock – the experiment carried out by observer A is measured by clock A, whereas B's experiment is measured by clock B. The time measured by the same clock is called *proper time* in relativity. In terms of spacetime proper time is a length of a timelike worldline. Indeed, as seen in Fig. 5.10 the worldtubes of clocks are "time rulers" in spacetime since their length is measured in seconds,

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[7] In Fig. 5.9 the inclined "cross section," which represents the different three-dimensional meter stick measured by B, appears longer, not shorter, because a fact in the pseudo-Euclidean geometry of spacetime is represented on the Euclidean surface of the paper.