A mathematical formalism in physics is not "just a matter of description"

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Abstract

Can metatheoretical misconceptions be ultimately responsible for the lack of breakthroughs in fundamental physics in recent decades? The answer outlined in the essay is yes. First I discuss such a misconception – that mathematics in physics is merely a description and therefore even fundamental mathematical entities (such as a manifold) do not represent counterparts in the physical world. Then I examine an instance of this misconception – that the four-dimensional manifold in relativity is only "an abstract four-dimensional mathematical continuum" – and summarize Minkowski's arguments that this four-dimensional manifold does represent a real four-dimensional world (spacetime). Finally, I discuss several negative implications of this misconception for the advancement of fundamental physics, including one which makes it impossible even to identify a radical (but not inconceivable) reason for the unsuccessful attempts to create a theory of quantum gravity.

1 Introduction

There has been no major breakthrough in fundamental physics in the last several decades as revolutionary as the theory of relativity and quantum mechanics despite the unprecedented advancements in applied physics and technology and despite the efforts of many brilliant physicists. It is not unthinkable to assume that this almost desperate situation may be caused by some metatheoretical misconceptions, not by the lack of sufficient experimental evidence and talented physicists.

I think misconceptions that are potentially most damaging for the advancement of fundamental physics concern the issue of the nature of physical theories and particularly its two main components:

1. There is no expiration date for physical theories whose predictions have been experimentally confirmed. Such theories will never be proven wrong in their domains of applicability where they were tested (for example, a thousand years from now bridges will still be built by employing the Newtonian mechanics). No future experiments can challenge such theories in the areas where their predictions were experimentally tested because experiments do not contradict one another. Any new theory containing the

domain of applicability of the old one will be a representation of the world with a better "resolution" and will not contradict the basic features of that domain captured by the old theory.

2. Fundamental mathematical entities (such as points, manifolds, etc.) in confirmed physical theories represent counterparts in the physical world (macroscopic particles are represented by points, space and time are represented by manifolds in Newtonian physics, etc.). Determining which mathematical entities have counterparts in the external world is indeed quite challenging partly because it is a bit atypical task for physicists since it does not involve calculations.

Properly understanding these components of the issue of the nature of physical theories is a necessary condition for the steady advancement of fundamental physics. There are two reasons for this. First, these two subissues outline how to extract foundational knowledge from the existing physical theories. Such knowledge will never be disproved by any experiments in the future namely because they cannot contradict the experiments that already confirmed the predictions of the existing theories, which are part of the foundational knowledge. Second, rigorous analyses of this knowledge may provide some clues about the direction in which the next breakthrough in fundamental physics should be sought (as we will see in Section 3).

Although the incorporation of the existing foundational knowledge into new theories has been so far de facto used as a necessary condition for their acceptance, it has not been explicitly explained

- what exactly is the foundational physical knowledge accumulated so far,
- how such knowledge should be extracted from the already accepted theories, and
- that any attempt to deepen our understanding of the physical world should start with a thorough analysis of the foundational knowledge explicitly aimed at identifying implications that may lead to a new breakthrough.

I will discuss a specific misconception to demonstrate that metatheoretical misconceptions about the nature of physical theories can delay the advancement of fundamental physics and can prevent even great scientists from making discoveries (a sad example is Poincaré who appears to have realized first that the Lorentz transformations are rotations in a four-dimensional mathematical space with time as the fourth dimension, but his conventionalism prevented him from accepting that this four-dimensional space represented a real four-dimensional world; Minkowski was free from the misconception that choosing a mathematical formalism is a matter of convention and made the discovery).

A misconception – that a physical phenomenon can be described *equally* by different theories because *"it is just a matter of description"* – effectively rules out the need for foundational knowledge and therefore hampers our understanding of the world and negatively affects the advancement of fundamental physics.

As there are a lot examples in physics that a given phenomenon can be described by different mathematical models, it is clear that part of the fascinating art of doing physics is to determine whether different theories are indeed simply different descriptions of the same physical phenomena (as is the case with the three representations of classical mechanics – Newtonian, Lagrangian, and Hamiltonian), or *only one* of the theories competing to describe and explain given physical phenomena is the correct one (as is the case with general relativity, which identifies gravity with the non-Euclidean geometry of spacetime, and other theories, which regard gravity as a force).

In Section 2 I will examine an instance of this misconception – that the four-dimensional manifold in relativity is only an abstract four-dimensional continuum – and will summarize Minkowski's arguments that this four-dimensional manifold represents a real four-dimensional world. In Section 3 I will discuss several negative implications of this misconception for the advancement of fundamental physics.

2 Is spacetime nothing more than "an abstract four-dimensional mathematical continuum"?

I see it, but I don't believe it
Cantor on his own discovery [3]

I find it inexplicable that, over a hundred years after Minkowski's talk "Space and and Time" in 1908, even physicists hold the experimentally unsupported view[1] that the concept of spacetime (represented by a four-dimensional manifold) is only an "abstract bookkeeping structure" [1], which is a successful *description* of the world. In other words, spacetime is nothing more than "an abstract four-dimensional mathematical continuum" [1]. Therefore, on this view, the concept of spacetime does not imply "that we inhabit a world that is such a four- (or, for some of us, ten-) dimensional continuum" [1].

Such views (see also [2]) are difficult to explain because they merely ignore Minkowski's arguments that the notion of spacetime (representing a real four-dimensional world) was forced upon as by the experimental evidence – that motion with constant velocity cannot be experimentally detected (i.e. that there is no difference between rest and uniform motion along a straight line). I think the phenomenon of ignoring the arguments (and the experimental evidence on which the arguments are based) for the reality of spacetime perhaps itself needs a special study and clarification[2] because physicists know well that an argument (especially if it is firmly supported by the experimental evidence) must be faced, not merely ignored.

[1]In addition to not being backed by experiment, the problem with this view is that, as we will see in Section 3, it is an unproductive one since it makes it impossible even to identify what the implications of a real spacetime are.

[2]I think the reason for ignoring these arguments is not scientific, because those arguments are merely treated as nonexistent. Quite possibly, the assertion that the physical world is four-dimensional is regarded as outrageously and self-evidently false, because of the colossally counter-intuitive nature of such a world and because of its huge implications for virtually all aspects of our lives. Perhaps such a reaction to arguments for disturbingly counter-intuitive new discoveries was best shown by Cantor in a letter to Dedekind in 1877 where he commented on the way he viewed one of his own major results (the one-to-one correspondence of the points on a segment of a line with (i) the points on an indefinitely long line, (ii) the points on a plane, and (iii) the points on any multidimensional mathematical space) – "I see it, but I don't believe it" [3].

Let me first summarize Minkowski's arguments that the concept of spacetime does represent a real four-dimensional world and then I will show that *the relativistic experimental evidence would be impossible if the world were three-dimensional* (for more detailed analysis see [4]-[6]).

On September 21, 1908 Minkowski began his famous lecture "Space and Time" by announcing the revolutionary view of space and time, which he deduced from experimental physics by successfully decoding the profound message hidden in the failed experiments to discover absolute motion [8, p. 111]:

> The views of space and time which I want to present to you arose from the domain of experimental physics, and therein lies their strength. Their tendency is radical. From now onwards space by itself and time by itself will recede completely to become mere shadows and only a type of union of the two will still stand independently on its own.

Minkowski made it exceedingly clear that it was an *experimental* fact that absolute motion and absolute rest cannot be discovered:

> "All efforts directed towards this goal, especially a famous interference experiment of Michelson had, however, a negative result" [8, p. 116]
>
> "In light of Michelson's experiment, it has been shown that, as Einstein so succinctly expresses this, the concept of an absolute state of rest entails no properties that correspond to phenomena" [10].

Then he revealed the profound physical meaning of the experimental failure to discover absolute motion. Minkowski showed *why* the time t of a stationary observer and the time t', which Lorentz introduced (as "an auxiliary mathematical quantity" [11]) to explain formally (mathematically) that failure calling it the *local time* of a moving observer (whose x' axis is along the x axis of the stationary observer), should be treated equally (which Einstein simply *postulated* in his 1905 paper) [8, p. 114]:

> One can call t' time, but then must necessarily, in connection with this, define space by the manifold of three parameters x', y, z in which the laws of physics would then have exactly the same expressions by means of x', y, z, t' as by means of x, y, z, t. Hereafter we would then have in the world no more *the* space, but an infinite number of spaces analogously as there is an infinite number of planes in three-dimensional space. Three-dimensional geometry becomes a chapter in four-dimensional physics. You see why I said at the beginning that space and time will recede completely to become mere shadows and only a world in itself will exist.

Minkowski's insistence that the new views of space and time "arose from the domain of experimental physics" now becomes obvious – the arguments that many times imply many spaces as well, which in turn implies that the world is four-dimensional, are deduced unambiguously from the *experiments* that gave rise to the principle of relativity – physical laws are the same in all inertial reference frames (i.e. that it is impossibile to discover absolute uniform motion and absolute rest). Indeed, all physical phenomena look in the same way to two observers A and B in uniform relative motion (so they cannot tell who is moving as the experimental evidence proved) *because* A and B have different times (as Lorentz formally proposed, Einstein postulated and Minkowski explained) and different spaces (as Minkowski first pointed out) – each observer performs experiments in his own space and time and for this reason the physical phenomena look in the same way to A and B, that is, physical laws are the same for A and B (e.g. the speed of light is the same for them since each observer measures it in his own space by using his own time). This explanation of the profound meaning of the principle of relativity, extracted from experimental physics, makes the non-existence of absolute motion and absolute rest quite evident – absolute motion and absolute rest do not exist since they are defined with respect to an absolute (i.e., *single*) space, but such a single space does not exist in the world; all observers in relative motion have their own spaces and times.

Minkowski did not state it explicitly that the experiments (on which the principle of relativity is based) would be impossible if the world were three-dimensional (i.e., if there existed *one* space and *one* time). I think he did not comment on that because he perhaps thought, as a mathematician, that it was quite obvious – if the four-dimensional world (spacetime) did not exist, there would exist *one* space and *one* time; so *space and time would be absolute*, which implies that absolute motion and absolute rest would be legitimate features of the world.

Another argument that relativity is impossible in a three-dimensional world is Minkowski's explanation of the deep physical meaning of length contraction (which is the accepted correct explanation). His explanation is depicted in Fig. 1 of his paper "Space and Time" whose right-hand part is reproduced here as Fig. 1.

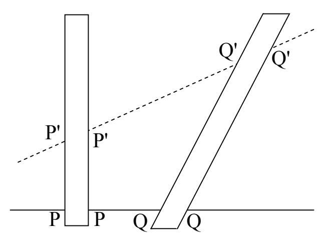

Figure 1: The right-hand part of Minkowski's Fig. 1 in his paper "Space and Time"

Note that the essence of his explanation is that the relativistic length contraction of a macroscopic body is a manifestation of the *reality* of the body's worldline or, more precisely, of the body's worldtube (since the body is spatially extended). Minkowski considered two bodies in uniform relative motion represented by their worldtubes as shown in Fig. 1. Consider the body represented by the vertical worldtube. The three-dimensional cross-section PP, resulting from the intersection of the body's worldtube and the space (represented by the horizontal line in Fig. 1) of an observer at rest with respect to the body, is the body's proper length. The three-dimensional cross-section $P'P'$, resulting from the intersection of the body's worldtube and the space (represented by the inclined dashed line) of an observer at rest with respect to the second body (represented by the inclined worldtube), is the relativistically contracted length of the body measured by that observer (the cross-section $P'P'$ only appears longer than PP because a fact of the pseudo-Euclidean geometry of spacetime is represented on the Euclidean surface of the page).

I believe it is clear why *the worldtube of a body must be real in order that length contraction be possible*, that is, why length contraction is impossible in a three-dimensional world – assume that the worldtube of the body did not exist as a four-dimensional object and were nothing more than an abstract geometrical construction; then, what would exist would be a single three-dimensional body, represented by the proper cross-section PP, and both observers would measure the *same* three-dimensional body of the *same* length. Therefore, not only would length contraction be *impossible*, but relativity of simultaneity would be also impossible since a spatially extended three-dimensional object is defined in terms of *simultaneity*[3] – all parts of a body taken *simultaneously* at a given moment – and as both observers in relative motion would measure the same three-dimensional body (represented by the cross-section PP) they would share the *same* class of simultaneous events in contradiction with relativity.

To understand fully why length contraction of a macroscopic body would be impossible if the body's worldtube were not real (that is, if the world were three-dimensional) let us examine a thought experiment which visualizes Minkowski's explanation of length contraction even further [5].

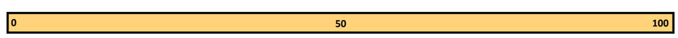

Figure 2: An ordinary meter stick.

I think this thought experiment demonstrates as clearly as possible that length contraction of a meter stick would be impossible if the meter stick existed as a three-dimensional body (not a worldtube). An ordinary meter stick (Fig. 2) is at rest with respect to an observer A. What is shown in Fig. 2 is what we perceive and take for granted that it is what really exists. According to Minkowski, however, the meter stick exists equally at all moments of its history and what is ultimately real is the worldtube of the meter stick as shown in Fig. 3.

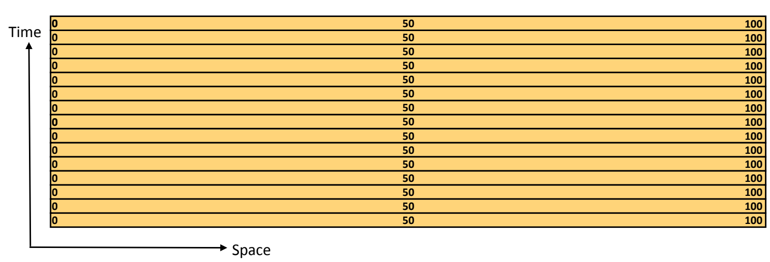

Figure 3: The worldtube of the meter stick.

Assume that another meter stick at rest in another observer's (observer B's) reference frame moves relative to the first one at a distance 1 mm above it. Let us assume

[3]Therefore, while measuring the *same* body, the two observers in Fig. 1 measure *two* three-dimensional bodies represented by the cross-sections PP and $P'P'$ (this relativistic situation will not be truly paradoxical only if what is meant by "the same body" is the body's worldtube).

that at event M the middle point of B's meter stick is instantaneously above the middle point of A's meter stick. Lights are installed inside A's meter stick, which change their color *simultaneously* at every instant in A's frame. At the event of the meeting M all lights are red in A's frame. At all previous moments (before the meeting at M) all lights were green. At all moments after the meeting all lights would be blue. When A and B meet at event M this event is present for both of them. At that moment all lights of A's meter stick will be *simultaneously red for* A. In other words, the present meter stick for A is red (that is, all parts of A's meter stick, which exist simultaneously for A, are red). All moments before M, when all lights of the meter were green, are past for A, whereas all moments when the meter stick will be blue are in A's future.

Figure 4: Relativistically contracted meter stick measured by observer B.

Imagine that B's meter stick contains cameras, instead of lights, at every point along its length. At the event of the meeting M all cameras take snapshots of the parts of A's meter stick which the cameras face. All snapshots are taken simultaneously in B's reference frame. Even without looking at the pictures taken by the cameras it is clear that not all pictures will show a red part of A's meter stick, because what is simultaneous for A is not simultaneous for B.

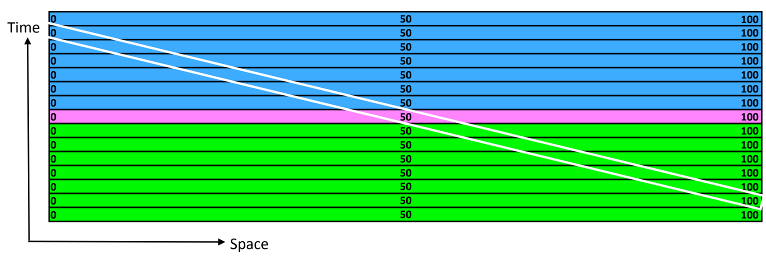

Figure 5: The worldtube of the meter stick with different colors.

When the picture of A's meter stick is assembled from the pictures of all cameras it would show two things as depicted in Fig 4 – (i) A's meter stick photographed by B is shorter, and (ii) only the middle part of the picture of A's meter stick is red; half is green and the other half is blue. So what is past (green), present (red), and future (blue) for A exists *simultaneously* as present for B. But note – *this is only possible if the meter stick is the worldtube as shown in Fig.* 5. The instantaneous space of B corresponding to the event M intersects the worldtube of the meter stick at an angle and the resulting three-color "cross section" is what is measured by B – a different three-dimensional meter stick, which is shorter[4] than the meter stick measured by A.

What should be stressed as strongly as possible is that not only length contraction as a theoretical prediction of relativity would be impossible if the world were three-dimensional, but the experiments which confirmed it would be also impossible.[5] And not only length contraction, but also time dilation, the twin paradox effect and the

[4]In Fig. 5 the inclined "cross section," which represents the different three-dimensional meter stick measured by B, appears longer, not shorter, as explained above.

[5]The muon experiment effectively tested length contraction experimentally along with time dila-

experiments that confirmed them would be also impossible in a three-dimensional world [4].

3 Negative implications of the misconception that spacetime is not real

The misconception that spacetime is only "an abstract four-dimensional mathematical continuum" and does not represent a real four-dimensional world has the potential to delay significantly the research in fundamental physics since it makes it impossible even to identify what the implications of a real spacetime are and whether they can shed some light on how open questions in physics might be resolved.

The main negative implication of this misconception is the missed opportunity to explore fully Minkowski's idea of geometrization of physics (even Einstein's general relativity did not go far enough). After Minkowski realized that four-dimensional physics was in fact spacetime geometry since all particles which *appear* to move in space and last in time are in reality a forever given web of the particles' worldlines in spacetime, he outlined his program [8, p. 112]: "The whole world presents itself as resolved into such worldlines, and I want to say in advance, that in my understanding the laws of physics can find their most complete expression as interrelations between these worldlines."

Here I will list three negative implications of the misconception that spacetime is only a mathematical concept without a counterpart in the world:

1. Missed opportunity to examine whether inertia arises from a four-dimensional stress in the deformed worldtube of an accelerating macroscopic body (the origin of such a stress can be traced down all the way to the quantum level where the inertia of quantum objects might be a result of distortion of the fundamental fields cased by the elementary particles' acceleration [6], [4, Chapter 9]).

2. Missed opportunity to identify a radical but legitimate explanation of the unsuccessful attempts to create a theory of quantum gravity – that gravity might not be a physical interaction – which does not appear to have been examined so far. Such a stunning possibility appears to follow naturally from Minkowski's program of regarding four-dimensional physics as spacetime geometry applied to a real spacetime ([6], [13]). If Einstein had examined fully the implications of Minkowski's program, he would have most probably considered this possibility and might have concluded that gravitational phenomena are not caused by gravitational interaction in general relativity since they are fully explained in the theory without the need to assume the existence of gravitational interaction: what has the appearance of gravitational attraction involves only *inertial* (*interaction-free*) motion and is indeed nothing more than a mere manifestation of the non-Euclidean geometry of spacetime. Had he lived longer, Minkowski himself might have arrived at this radical possibility. In 1921 Eddington even mentioned it explicitly – "gravitation as a separate agency becomes unnecessary" [14].

tion: "in the muon's reference frame, we reconcile the theoretical and experimental results by use of the length contraction effect, and the experiment serves as a verification of this effect" [12].

3. Missed opportunity to try to clarify both the nature of the quantum objects and the apparent incompatibility between the probabilistic behaviour of quantum objects and the forever given spacetime. As quantum objects are not worldlines in spacetime, it could have been examined whether they might be more complex structures in spacetime (for a conceivable example see [4, Chap. 10] and the references therein). As an illustration that spacetime can naturally accommodate probability, imagine that the probabilistic behaviour of the quantum object is a manifestation of a *probabilistic distribution of the quantum object itself in the forever given spacetime* – an electron, for instance, can be thought of as an ensemble of the points of its "disintegrated" worldline which are scattered in the spacetime region where the electron wavefunction is different from zero. Had Minkowski lived longer, he might have described such a probabilistic spacetime structure by another[6] mystical expression – predetermined probabilistic phenomena.

Conclusion

In this essay I have tried to demonstrate that mathematics in physics is not merely a description and that part of the exciting art of doing physics is to determine which mathematical entities have counterparts in the physical world. Also, despite that the issue of the nature and role of the mathematical formalism in physics is a metatheoretical issue, physicists should deal with it because misconceptions about this issue might delay the advancement of fundamental physics.

[6]See [8, p. 122]: "Thus the essence of this postulate can be expressed mathematically very concisely in the mystical formula: $3 \cdot 10^5\ km = \sqrt{-1}\ seconds$."

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