Physics as Spacetime Geometry

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Abstract

As there have been no major advancements in fundamental physics in the last decades it seems reasonable to reexamine the major explicit and especially implicit assumptions in fundamental physics to ensure that all logically possible research directions are identified. The purpose of this chapter is to outline such a direction. Minkowski's program of regarding four-dimensional physics as spacetime geometry is rigorously and consistently employed to the already geometrized general relativity with the most stunning implication that gravitational phenomena are fully explained in the theory without the need to assume that they are caused by gravitational interaction. Then the real open question in gravitational physics seems to be how matter curves spacetime, not how to quantize the apparent gravitational interaction. In view of the difficulties encountered by quantum gravity, even the radical option that gravity is not a physical interaction deserves careful scrutiny due to its potential impact on fundamental physics as a whole. The chapter discusses the possible implications of this option for the physics of gravitational waves and for quantum gravity and ends with an example where regarding physics as spacetime geometry provides a straightforward explanation of a rather subtle issue in relativity – propagation of light in non-inertial reference frames.

1 Foundational Knowledge and Reality of Spacetime

Minkowski's program of regarding four-dimensional physics as spacetime geometry is often viewed as just a more convenient description of physical phenomena. However, I think Minkowski's program is crucially important for fundamental physics; hence, the program and its implications should be rigorously examined for the following reason. The identification of four-dimensional physics with the geometry of spacetime presupposes that spacetime represents a real four-dimensional world as Minkowski insisted since physics cannot be geometry of something abstract (here we again face the challenging question of whether a mathematical formalism is only a convenient description of physical phenomena or reveals true features of the physical world). However, the status of spacetime has been unresolved and this might turn out to be ultimately responsible for the failure to create a quantum theory of gravity so far, and possibly even for the fact that in the last several decades there has been no major breakthrough as revolutionary as the theory of relativity and quantum mechanics despite the unprecedented advancements in applied physics and technology and despite the efforts of many brilliant physicists.

It is not inconceivable to assume that the present state of fundamental physics may be caused by some metatheoretical problems, not by the lack of sufficient experimental evidence and talented physicists. I think the major metatheoretical reason for most difficulties in contemporary fundamental physics, and particularly for not dealing with the status of spacetime, is underestimating the necessity to identify explicitly which elements of our theories adequately represent elements of the physical world. Such reliable knowledge about the world is a necessary condition for the smooth advancement of fundamental physics since it forms the foundation on which new theories are built. To ensure that such foundational knowledge will never be revised as our understanding of the world deepens, that knowledge should be rigorously and *unambiguously* extracted from the *experimental* evidence. As experiments do not contradict one another no future discoveries can challenge the accumulated foundational knowledge. In 1909 Max Planck expressed the idea of foundational knowledge (whose elements he properly called invariants) perhaps in the best possible way [1]:

> The principle of relativity holds, not only for processes in physics, but also for the physicist himself, in that a fixed system of physics exists in reality only for a given physicist and for a given time. But, as in the theory of relativity, there exist invariants in the system of physics: ideas and laws which retain their meaning for all investigators and for all times, and to discover these invariants is always the real endeavor of physical research. We shall work further in this direction in order to leave behind for our successors where possible – lasting results. For if, while engaged in body and mind in patient and often modest individual endeavor, one thought strengthens and supports us, it is this, that we in physics work, not for the day only and for immediate results, but, so to speak, for eternity.

In close connection with the necessity for explicit foundational knowledge, it is worth stressing that a view, which some physicists are sometimes tempted to hold – that physical phenomena can be described *equally* by different theories (*"it is just a matter of description"*) – hampers our understanding of the world and negatively affects the advancement of fundamental physics since such a view effectively rules out the need for foundational knowledge. I hope all will agree that part of the art of doing physics is to determine whether different theories are indeed simply different descriptions of the same physical phenomena (as is the case with the three representations of classical mechanics – Newtonian, Lagrangian, and Hamiltonian), or *only one* of the theories competing to describe and explain given physical phenomena is the correct one (as is the case with general relativity, which identifies gravity with the non-Euclidean geometry of spacetime, and other theories, which regard gravity as a force).

Due to the unsettled status of spacetime, there are physicists who hold the experimentally unsupported view that the concept of spacetime is only a successful *description* of the world (an "abstract bookkeeping structure" [2]) and for this reason it is nothing more than "an abstract four-dimensional mathematical continuum" [2]. Therefore, on this view, the concept of spacetime does not imply "that we inhabit a world that is such a four- (or, for some of us, ten-) dimensional continuum" [2]. In addition to not being backed by experiment, the problem with this view is that it is an unproductive one since it makes it impossible even to identify what the implications of a real spacetime are. As those implications might turn out to be necessary for the advancement of fundamental physics, Section 2 deals with the essence of the spacetime concept (the reality of spacetime, i.e. that the world is four-dimensional) and argues that the relativistic experimental evidence provides strong support for it, which allows us to regard the reality of spacetime as an important piece of foundational knowledge. Section 2 also examines how Minkowski's program of geometrization of physics sheds additional light on Einstein's geometrization of gravity and suggests that gravitational phenomena are not caused by gravitational interaction since those phenomena are fully explained in general relativity without the need of gravitational interaction. The implications of this possibility for the search for gravitational waves and for quantum gravity are discussed in the last part of the section. Section 3 demonstrates how taking the reality of spacetime explicitly into account makes it self-evident why the propagation of light in non-inertial reference frames in flat and curved spacetimes is anisotropic [3].

2 Four-dimensional Physics as Spacetime Geometry

In the beginning of this section I will summarize what I think is the unequivocal experimental evidence which indicates that the concept of spacetime does represent a real four-dimensional world. If the arguments convincingly show (as I believe they do) that *the relativistic experimental evidence would be impossible if the world were three-dimensional*, then the reality of spacetime (i.e., the assertion that the world is four-dimensional) is indeed a major piece of foundational knowledge.

It was Minkowski who initially extracted this foundational knowledge from the experimental evidence that supported the relativity principle. On September 21, 1908 he began his famous lecture "Space and Time" by announcing the revolutionary view of space and time, which he deduced from experimental physics by successfully decoding the profound message hidden in the failed experiments to discover absolute motion [4, p.111]:

> The views of space and time which I want to present to you arose from the domain of experimental physics, and therein lies their strength. Their tendency is radical. From now onwards space by itself and time by itself will recede completely to become mere shadows and only a type of union of the two will still stand independently on its own.

Minkowski repeatedly stressed the *experimental* fact that absolute motion and absolute rest cannot be discovered:

> "All efforts directed towards this goal, especially a famous interference experiment of Michelson had, however, a negative result" [4, p. 116]
>
> "In light of Michelson's experiment, it has been shown that, as Einstein so succinctly expresses this, the concept of an absolute state of rest entails no properties that correspond to phenomena" [6].

Minkowski had apparently felt that the experimental evidence supporting Galileo's principle of relativity (absolute motion with constant velocity cannot be discovered through mechanical experiments) and the failed experiments (involving light beams) to detect the Earth's motion contained some hidden information about the physical world that needed to be decoded. That is why he had not been satisfied with the principle of relativity which merely *postulated* that absolute motion and absolute rest did not exist. To decode the hidden information, Minkowski first examined (as a mathematician) the fact that "The equations of Newtonian mechanics show a twofold invariance" [4, p. 111]. As each of the two invariances represents a certain group of transformations for the differential equations of mechanics Minkowski noticed that the second group (representing invariance with respect to uniform translations, i.e. Galileo's principle of relativity) leads to the conclusion that the "time axis can then be given a completely arbitrary direction in the upper half of the world $t > 0$." This strange implication made Minkowski ask the question that led to the new view of space and time: "What has now the requirement of orthogonality in space to do with this complete freedom of choice of the direction of the time axis upwards?" [4, p. 111].

In answering this question Minkowski showed *why* the time t of a stationary observer and the time t', which Lorentz introduced (as "an auxiliary mathematical quantity" [8]) calling it the *local time* of a moving observer (whose x' axis is along the x axis of the stationary observer), should be treated equally (which Einstein simply *postulated* in his 1905 paper) [4, p. 114]:

> One can call t' time, but then must necessarily, in connection with this, define space by the manifold of three parameters x', y, z in which the laws of physics would then have exactly the same expressions by means of x', y, z, t' as by means of x, y, z, t. Hereafter we would then have in the world no more *the* space, but an infinite number of spaces analogously as there is an infinite number of planes in three-dimensional space. Three-dimensional geometry becomes a chapter in four-dimensional physics. You see why I said at the beginning that space and time will recede completely to become mere shadows and only a world in itself will exist.

The profound implication of "the requirement of orthogonality in space" is evident in the beginning of this quote – as t and t' are two different times it necessarily follows that two different spaces must be associated with these times since each space is orthogonal to each time axis. Minkowski easily saw the obvious for a mathematician fact that different time axes imply different spaces and remarked that "the concept of space was shaken neither by Einstein nor by Lorentz" [4]. Then, as the quote demonstrates, Minkowski had immediately realized that many spaces and times imply that the world is four-dimensional with *all* moments of time forming the fourth dimension (Poincaré showed before Minkowski that the Lorentz transformations can be regarded as rotations in a four-dimensional space with time as the fourth axis but, unlike Minkowski, he did not believe that that mathematical space represented anything in the world; see the Introduction of [5], particularly pages 19-23, and the reference therein).

Minkowski excitedly announced the new views of space and time since he clearly recognized that their strength comes from the fact that they "arose from the domain of experimental physics" – the arguments that many times imply many spaces as well, which in turn implies that the world is four-dimensional, are deduced unambiguously from the *experiments* that confirmed the principle of relativity (i.e. the impossibility to discover absolute uniform motion and absolute rest). Indeed, all physical phenomena look in the same way to two observers A and B in uniform relative motion (so they cannot tell who is moving as the experimental evidence proved) *because* A and B have different times (as Lorentz formally proposed, Einstein postulated and Minkowski explained) and different spaces – each observer performs experiments in his own space and time and for this reason the physical phenomena look in the same way to A and B (e.g. the speed of light is the same for them since each observer measures it in his own space by using his own time). This explanation of the profound meaning of the principle of relativity, extracted from experimental physics, makes the non-existence of absolute motion and absolute rest quite evident – absolute motion and absolute rest do not exist since they are defined with respect to an absolute (single) space, but such a single space does not exist in the world; all observers in relative motion have their own spaces and times.

The most direct way to evaluate Minkowski's confidence in the strength of the new views of space and time and his insistence that they were deduced from experimental physics is to assume, for the sake of the argument, that spacetime is nothing more than "an abstract four-dimensional mathematical continuum" [2] and that the physical world is three-dimensional. Then there would exist a *single* space (since a three-dimensional world presupposes the existence of one space), which as such would be absolute (the same for all observers). As a space constitutes a class of simultaneous events (the space points at a given moment), a single (absolute) space implies absolute simultaneity and therefore absolute time as well. Hence a three-dimensional world allows *only* absolute space and absolute time in contradiction with the experimental evidence that uniform motion with respect to the absolute space cannot be discovered as encapsulated in the principle of relativity. Minkowski's realization that the world must be four-dimensional in order that absolute motion and rest do not exist naturally explains his dissatisfaction with the principle of relativity, which postulates, but does not explain the non-existence of absolute motion and rest [4, p. 117]:

> I think the word *relativity postulate* used for the requirement of invariance under the group G_c is very feeble. Since the meaning of the postulate is that through the phenomena only the four-dimensional world in space and time is given, but the projection in space and in time can still be made with a certain freedom, I want to give this affirmation rather the name *the postulate of the absolute world*.

In addition to Minkowski's arguments, I would like to stress what I consider to be a fact that special relativity and particularly the *experiments*, which confirmed the kinematical relativistic effects, are *impossible* in a three-dimensional world. I think each of the arguments listed below taken even alone is sufficient to demonstrate that.

- Relativity of simultaneity is impossible in a three-dimensional world – as a three-dimensional world (like a three-dimensional space) is a class of simultaneous events (everything that exists simultaneously at the present moment), if the physical world were three-dimensional, there would exist a single class of simultaneous events; therefore simultaneity would be absolute since all observers in relative motion would share the same three-dimensional world and therefore the same class of simultaneous events.

- Since length contraction and time dilation are specific manifestations of relativity of simultaneity they are also impossible in a three-dimensional world. What is crucial is that the *experiments* which confirmed these relativistic effects would be impossible if the physical world were three-dimensional [10, Chap. 5]. Along with time dilation, the muon experiment (see, for example, [9, p. 103]) effectively tested length contraction experimentally as well – "in the muon's reference frame, we reconcile the theoretical and experimental results by use of the length contraction effect, and the experiment serves as a verification of this effect" [9, p. 104].

- The twin paradox effect and the experiments that confirmed it are also impossible in a three-dimensional world [10, Chap. 5].

A valuable concrete example of why special relativity is impossible in a three-dimensional world is Minkowski's explanation of the deep physical meaning of length contraction as depicted in Fig. 1 of his paper "Space and Time" whose right-hand part is reproduced here as Fig. 1.

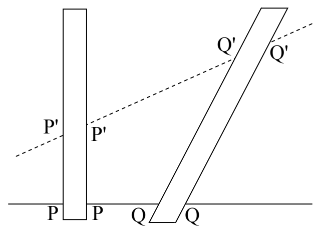

Figure 1: The right-hand part of Fig. 1 in Minkowski's paper "Space and Time"

The essence of his explanation (which is the accepted correct explanation) is that the relativistic length contraction of a body is a manifestation of the reality of the body's worldline or rather worldtube (for a spatially extended body). Minkowski considered two bodies in uniform relative motion represented by their worldtubes as shown in Fig. 1. To see clearly why *the worldtube of a body must be real in order that length contraction be possible*, consider the body represented by the vertical worldtube. The three-dimensional cross-section PP, resulting from the intersection of the body's worldtube and the space (represented by the horizontal line in Fig. 1) of an observer at rest with respect to the body, is the body's proper length. The three-dimensional cross-section $P'P'$, resulting from the intersection of the body's worldtube and the space (represented by the inclined dashed line) of an observer at rest with respect to the second body (represented by the inclined worldtube), is the relativistically contracted length of the body measured by that observer (the cross-section $P'P'$ only appears longer than PP because a fact of the pseudo-Euclidean geometry of spacetime is represented on the Euclidean surface of the page). Note that while measuring the *same* body, the two observers measure *two* three-dimensional bodies represented by the cross-sections PP and $P'P'$ in Fig. 1 (this relativistic situation will not be truly paradoxical only if what is meant by "the same body" is the body's worldtube).

In order to judge the argument that length contraction is impossible in a three-dimensional world, assume that the worldtube of the body did not exist as a four-dimensional object and were nothing more than an abstract geometrical construction. Then, what would exist would be a single three-dimensional body, represented by the proper cross-section PP, and both observers would measure the *same* three-dimensional body of the *same* length. Therefore, not only would length contraction be *impossible*, but relativity of simultaneity would be also impossible since a spatially extended three-dimensional object is defined in terms of *simultaneity* – all parts of a body taken *simultaneously* at a given moment – and as both observers in relative motion would measure the same three-dimensional body (represented by the cross-section PP) they would share the *same* class of simultaneous events in contradiction with relativity.

After Minkowski had successfully decoded the profound message hidden in the failed experiments to detect absolute uniform motion and absolute rest – that the world is four-dimensional – he had certainly realized that four-dimensional physics was in fact spacetime geometry since all particles which *appear* to move in space are in reality a forever given web of the particles' worldlines in spacetime. Then Minkowski outlined the program of geometrization of physics [4, p. 112]: "The whole world presents itself as resolved into such worldlines, and I want to say in advance, that in my understanding the laws of physics can find their most complete expression as interrelations between these worldlines." And before his tragic and untimely departure from this world on January 12, 1909 he started to implement this program as will be briefly discussed below. But let me first address a view which some physicists are sometimes tempted to hold – that we should not take the implications of special and general relativity too seriously because these theories cannot accommodate the probabilistic behaviour of quantum objects.

In fact, it is that view which should not be taken seriously for two reasons. First, as it is the *experiments* confirming the kinematical relativistic effects that would be impossible in a three-dimensional world, the reality of spacetime (the four-dimensionality of the world) must be treated with utmost seriousness. Since *experiments do not contradict one another* no future experiments can force us to abandon the view that the world is four-dimensional and that macroscopic bodies are worldtubes in this world. Second, the fact that elementary particles are not worldlines in spacetime only indicates *what they are not* and in no way tells us something against the reality of spacetime. Elementary particles, or perhaps more appropriately quantum objects, might be more complex structures in spacetime (for a conceivable example see [10, Chap. 10] and the references therein). As an illustration that spacetime can accommodate probability perfectly well, imagine that the probabilistic behaviour of the quantum object is merely a manifestation of a *probabilistic distribution of the quantum object itself in the forever given spacetime* – an electron, for instance, can be thought of (for the sake of the argument that spacetime structures can be probabilistic as well) as an ensemble of the points of its "disintegrated" worldline which are scattered in the spacetime region where the electron wavefunction is different from zero. Had Minkowski lived longer he might have described such a probabilistic spacetime structure by the mystical expression "predetermined probabilistic phenomena."

I think the very fact that the status of spacetime has not been firmly settled for over a hundred year deserves special attention since it may provide some valuable lessons for the future of fundamental physics. Indeed, it is *logically* inexplicable why Minkowski's effective arguments for the reality of spacetime have been merely ignored (they have not been refuted); as we saw above his arguments taken alone demonstrate that the world must be four-dimensional in order that special relativity and the experimental evidence which tested its kinematical effects be possible. It appears the reason for ignoring the arguments for the reality of spacetime are not scientific; the reason does not seem to be even rational since those arguments are merely regarded as nonexistent. Quite possibly, such an attitude towards the nature of spacetime is caused by the temptation to regard the claim that the physical world is four-dimensional as an outrageously and self-evidently false, because of the colossally counter-intuitive nature of such a world and because of its huge implications for virtually all aspects of our lives. Perhaps such a reaction to arguments for disturbingly counter-intuitive new discoveries was best shown by Cantor in a letter to Dedekind in 1877 where he commented on the way he viewed one of his own major results (the one-to-one correspondence of the points on a segment of a line with (i) the points on an indefinitely long line, (ii) the points on a plane, and (iii) the points on any multidimensional mathematical space) – "I see it, but I don't believe it" [7]. However, the nature of the world as revealed by the experimental evidence – no matter how counter-intuitive it may be – should be faced and should not be squeezed into our preset and deceivingly comfortable views about what exists.

Due to the unsettled status of spacetime so far, Minkowski's program of adequately treating four-dimensional physics as spacetime geometry has not been fully implemented. As a result, new discoveries leading to a deeper understanding of the world might have been delayed. A small example is the propagation of light in non-inertial reference frames – this issue could have been addressed and clarified immediately after Minkowski's four-dimensional formulation of special relativity. In the remaining part of this section I will discuss first Minkowski's initial steps of the implementation of his program of geometrization of physics and then will outline other unexplored implications of his program some of which may have significantly affected front line research programs in fundamental physics such as the search for gravitational waves and quantum gravity.

Generalization of inertial motion in special and general relativity. Minkowski generalized Newton's first law (of inertia) for the case of flat spacetime by noticing that a free particle, which is at (relative) rest or moves by inertia, is a straight timelike worldline. Then he pointed out that an accelerating particle is represented by a curved worldline. Here is how Minkowski described the three states of motion of a particle (corresponding to the worldlines a, b, and c in Fig. 2) [4, p. 115]:

> A straight worldline parallel to the t-axis corresponds to a stationary substantial point, a straight line inclined to the t-axis corresponds to a uniformly moving substantial point, a somewhat curved worldline corresponds to a non-uniformly moving substantial point.

As a straight timelike worldline represents inertial motion it immediately becomes clear why experiments have always failed to distinguish between a state of rest and a state of uniform motion – in both cases a particle is a *straight* worldline as seen in Fig. 2 (worldlines a and b) and there is clearly no distinction between two straight lines. In the figure the time axis of the reference frame is along worldline a and the particle represented by this worldline appears to be at rest in the reference frame. If the time axis of another reference frame is chosen along worldline b, the particle represented by that worldline

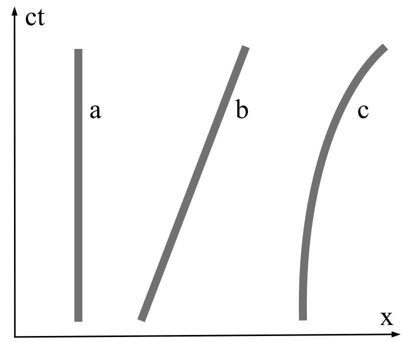

Figure 2: Worldlines a and b represent two particles – one at rest (a) and the other in uniform motion (b), whereas worldline c represents an accelerating particle.

will appear to be at rest in the new reference frame, whereas the first particle (represented by worldline a) will appear to be uniformly moving with respect to the second particle (since worldline a is inclined to the new time axis, i.e. inclined to worldline b). Minkowski seems to have been impressed by this elegant explanation of the experimental fact that rest and uniform motion cannot be distinguished (which is a more detailed *explanation* of the relativity principle) that he decided "to introduce this fundamental axiom: *With appropriate setting of space and time the substance existing at any worldpoint can always be regarded as being at rest*" [4, p. 115] .

Perhaps the most successful continuation of Minkowski's program of geometrization of physics is the generalization of inertial motion in general relativity. This is encapsulated in the geodesic hypothesis in general relativity, which states that the worldline of a free particle is a timelike *geodesic* in spacetime. This hypothesis is regarded as "a natural generalization of Newton's first law" [11, p. 110], that is, "a mere extension of Galileo's law of inertia to curved spacetime" [12, p. 178]. This means that *in general relativity a particle, whose worldline is geodesic, is a free particle which moves by inertia.*

Unfortunately, the important implications of this rare implementation of Minkowski's program have not been fully explored, which might have delayed the research in gravitational physics, particularly the initiation and advancement of a research program to reveal the mechanism of how matter curves spacetime. The immediate consequence of the geodesic hypothesis and its experimental confirmation by the fact that falling bodies do not resist their fall (a falling accelerometer, for example, reads zero resistance, i.e. zero absolute acceleration, since it measures acceleration through resistance) implies that the explanation of gravitational phenomena does not need the assumption of the existence of gravitational interaction. The reason is that as a falling body moves by inertia (since it does not resist its fall) no gravitational force is causing its fall, i.e., it is not subject to any interactions since inertial, i.e., non-resistant, motion by its very nature is *interaction-free* motion. The analysis of this consequence of the geodesic hypothesis naturally leads to the question of how matter curves spacetime in order to determine whether the Earth interacts gravitationally with a falling body through the curvature of spacetime. If there is such an interaction between the Earth and the body, there should exist *extra* stress-energy of the Earth not only to curve spacetime but to change the shape of the geodesic worldtube of the falling body (that change of shape makes it more "curved", but not deformed, as will be discussed below, which means that the worldtube of the falling body is geodesic and the body does not resist its fall). As we will see below this does not appear to be the case since the Einstein-Hilbert equation implies that no extra stress-energy is necessary to change the shape of the geodesic worldtube of a falling body – the same stress-energy of the Earth, for example, produces the same spacetime curvature no matter whether or not there are other bodies in the Earth's vicinity.

The importance of the experimental fact that falling bodies offer no resistance to their fall is that *it rules out* any alternative theories of gravity and any attempts to quantize gravity (by proposing alternative representations of general relativity aimed at making it amenable to quantization) that regard gravity as a *physical* field which gives rise to a gravitational *force* since they would contradict the experimental evidence. It should be particularly stressed that a gravitational force would be required to move particles downwards *only if* the particles *resisted* their fall, because *only then* a gravitational force would be needed to *overcome* that resistance.

In what sense is acceleration absolute in both special and general relativity? Minkowski's own implementation of his program to represent four-dimensional physics as spacetime geometry produced another important result – an unforeseen resolution of the debate over the status of acceleration, which was prompted by Newton's insistence that both acceleration and space are absolute (since acceleration is experimentally detectable, and therefore absolute, which implies that space is also absolute due to the apparently self-evident assumption that any acceleration is with respect to space).

Encouraged by the resolution of the centuries-old puzzle (that it is impossible to distinguish experimentally between rest and uniform motion) in terms of the geometry of spacetime – two particles, one at rest and the other in uniform motion, are both *straight* worldlines in spacetime – Minkowski almost certainly had immediately seen that another experimental fact – acceleration is experimentally detectable – also had an elegant explanation in terms of spacetime geometry: an accelerating particle is a *curved* worldline in spacetime. He expressed this observation by stressing that "Especially the concept of *acceleration* acquires a sharply prominent character" [4, p. 117].

Minkowski left this world less than four months after he gave his last and famous lecture "Space and Time" where he talked about that "sharply prominent character" of acceleration. He was not given the chance to develop further his ideas. But Minkowski succeeded in revealing the deep physical meaning of the distinction between inertial and accelerated motion: the absolute physical facts that inertial motion cannot be detected experimentally, whereas accelerated motion is experimentally detectable, correspond to two geometrical facts in spacetime – a particle moving by inertia is a *straight* worldline, whereas an accelerating particle is a *curved* worldline. Such an explanation of physical facts by facts of the geometry of spacetime is not only natural, but is the only explanation in a real four-dimensional world – in such a world, i.e. in spacetime, there are no three-dimensional particles which move inertially or with an acceleration; there is only a forever given network of straight and curved worldlines there.

The *absoluteness* (frame-independence) of acceleration and inertial motion is reflected in the *curvature* of the worldline of an accelerating particle and the *straightness* of the worldline of a particle moving by inertia, respectively, which are *absolute geometrical properties* of the particles' worldlines. So *acceleration is absolute not because a particle accelerates with respect to some absolute space, but because its worldline is curved*, which is a geometrical fact that is frame-independent (and indeed there is neither motion nor a distinguished space in spacetime). In the same way, inertial motion is absolute – in any reference frame the worldline of an inertial particle is straight. This deep understanding of inertial and accelerated motion in terms of the *shape* of particles' worldlines and *with no reference to space* nicely explains the apparent paradox that seems to have tormented Newton the most – both an inertial and an accelerating particle (appear to) move *in* space, but only the accelerating particle resists its motion. Below we will see that this nice explanation becomes beautiful when it is taken into account that the geometry of a real four-dimensional world is physical geometry which involves real physical objects – worldlines or rather worldtubes in the case of spatially extended bodies.

In general relativity (in curved spacetime) the absoluteness of inertial motion reflects the absolute (frame-independent) geometrical property of the worldline of a free particle to be geodesic. By analogy with the absoluteness of acceleration in flat spacetime, the absoluteness of acceleration in curved spacetime manifest itself in the fact that the worldline of a particle, whose curved-spacetime acceleration ($a^\mu = d^2x^\mu/d\tau^2 + \Gamma^\mu_{\alpha\beta}(dx^\alpha/d\tau)(dx^\beta/d\tau)$) is different from zero, is not geodesic – the worldline of such a particle is curved or, perhaps more precisely, deformed (intuitively, that deformation can be regarded as an additional curvature to the natural curvature of a geodesic worldline which is due to the curvature of spacetime itself; rigorously, in general relativity a geodesic is not curved, only non-geodesic worldlines are curved). There is a second acceleration in general relativity caused by geodesic deviation, which reflects two facts – that there are no straight worldlines and no parallel or rather congruent worldlines in curved spacetime. This acceleration is not absolute, but relative since it involves two geodesic worldlines (which are not deformed), whereas absolute acceleration involves a single non-geodesic worldline (which is deformed).

Regarding four-dimensional physics as spacetime geometry easily refutes Mach's view of the relativity of acceleration [13]. Two consequences of this view discussed by Mach himself [15] are (i) the equivalence of rotation and translation and (ii) the relativity of rotation, which implied (as Mach stated) the equivalence of the Ptolemaic and the Copernican models of our planetary system. It is clear that rotation and translation are distinct in spacetime – the worldline of a particle moving translationally is either a straight line in flat spacetime (when the particle moves uniformly) or a curved line (when the particle accelerates translationally), whereas the worldline of a rotating particle is a helix. This also explains why the Ptolemaic and the Copernican systems are not equivalent – the planets' worldlines are helixes around the worldline of the Sun. Another consequence of Mach's view is that if there were no other bodies in the Universe, one could not talk about the state of motion of a *single* particle since on Mach's view only motion *relative* to a body makes sense. In spacetime the situation is crystal clear – a single particle in the Universe is either a geodesic worldline (which means that the particle moves by inertia) or a deformed worldline (which means that the particle accelerates).

Inertia as another manifestation of the reality of spacetime. Had Minkowski lived longer he would have certainly noticed that his explanation of the absoluteness of accelerated and inertial motion in terms of the absolute geometrical properties of particles' worldlines (curvature and straightness, respectively) not only reflected the experimental (and therefore absolute, i.e. frame-independent) facts that an accelerating body resists its acceleration, whereas a particle moving by inertia offers no resistance to its motion, but could also *explain* these facts.

Two pieces of reliable knowledge about an accelerating body would have appeared naturally linked in the spacetime explanation of the absoluteness of acceleration – an accelerating body (i) resists its acceleration, and (ii) is represented by a curved (and therefore *deformed*) worldtube. Then taking into account the reality of the body's worldtube (relativistic length contraction would be impossible if the worldtube of the contracting body were not real as seen from Minkowski's explanation discussed above), would have led to the logically evident, but totally unexpected consequence of linking the two features of the accelerating body – the resistance an accelerating body offers to its acceleration could be viewed as originating from a four-dimensional stress in the deformed worldtube of the body. And it turns out that the *static* restoring force existing in the deformed worldtube of an accelerating body does have the form of the inertial force with which the body resists its acceleration [10, Chap. 9]. The origin of the static restoring force (i.e., the inertial force) can be traced down to the most fundamental constituents of matter – as an elementary particle is not a worldline in spacetime its inertia appears to originate from the *distorted* fields which mediate the particle's interactions (the distortion of the fields is caused by the particle's acceleration) [10, Chap. 9].

I guess, Minkowski would have been truly thrilled – inertia appears to be another manifestation of the four-dimensionality of the world (since only a *real* worldtube could resist its deformation) along with the other manifestations he knew then – length contraction and all experiments demonstrating that absolute uniform motion could not be detected (that is, that rest and uniform motion could not be distinguished experimentally).

With this insight into the origin of inertia implied by the reality of spacetime, the experimental distinction between accelerated and inertial motion finds a natural but counter-intuitive explanation – an accelerating body resists its acceleration since its worldtube is *deformed* and the static restoring force existing in the worldtube is interpreted as the inertial force, whereas a particle moving by inertia offers no resistance to its motion since its worldtube is *not deformed* – it is straight in flat spacetime and geodesic in curved spacetime – and therefore no restoring force exists in the particle's worldtube (which explains why inertial motion cannot be detected experimentally).

Why is the inertial force equivalent to the force of weight? The equivalence of the inertial force with which a particle resists its acceleration and the particle's weight (or the gravitational force acting on the particle in terms of the Newtonian gravitational theory) is best visualized by Einstein's thought experiments involving an accelerating elevator and an elevator on the Earth's surface. Assume that a particle is on the floor of an elevator whose acceleration a is equal to the acceleration due to gravity g. The particle exerts on the elevator's floor an inertial force with which it resists its acceleration (forced on it by the floor). When the same elevator with the same particle on its floor is on the Earth's surface, the particle exerts on the floor a force of the same magnitude which is called the particle's weight (in the Newtonian gravitational theory this force is regarded as the gravitational force acting on the particle). Einstein regarded the equivalence of the two forces as a manifestation of his principle of equivalence according to which the effects of accelerated motion and gravity cannot be distinguished *locally* in spacetime (i.e., for *small* distances and *short* periods of time). In other words, if an observer in a small elevator (i) measures the weight of a particle and (ii) studies for a short period of time its fall towards the floor of the elevator, he will be unable to determine from his measurements whether the elevator is accelerating with $a = g$ or it is on the Earth's surface.

Initially, Einstein *postulated* the equivalence of the inertial and gravitational forces as part of the principle of equivalence which was a crucial step in the creation of general relativity. Later, when Minkowski's representation of inertial and accelerated motion in spacetime was generalized for the case of curved spacetime it became possible to reveal the deep meaning of this equivalence and of the principle of equivalence itself – inertial and gravitational forces (and masses as will be discussed below) are equivalent since they both are inertial forces (and masses).

By the geodesic hypothesis in general relativity (confirmed by the experimental fact that falling bodies do not resist their fall), a particle falling towards the Earth's surface moves by inertia since its worldtube is geodesic (more precisely, the center of the particle's mass is a geodesic worldline). This means that the particle does not resist its motion in agreement with the fact that its worldtube is not deformed (since it is geodesic).

When the particle reaches the ground it is prevented from moving by inertia (i.e. prevented from falling) and the particle resists the change in its inertial motion. In other words, the particle on the ground is *accelerating* since it is forced by the Earth's surface to *change* its motion by inertia. This counter-intuitive fact – that a particle on the ground accelerates, whereas it is obviously at rest there – is naturally explained by the generalization of Minkowski's observation (see also [14, Chap. 9]) that in spacetime an accelerating particle is a curved (deformed) worldtube. Indeed in general relativity the acceleration of a particle at rest on the Earth's surface is the (first) curvature of the particle's worldtube [11, pp. 138, 177]. The worldtube of the falling particle is geodesic, but starting at the event at which the particle touches the ground, the particle's worldtube is *constantly* deformed by the huge worldtube of the Earth, which means that the particle on the ground is constantly accelerating (the particle's absolute acceleration is a manifestation of its deformed worldtube).

As the worldtube of the particle, when it is at rest on the ground, is deformed the static restoring force in the worldtube acts back on the Earth's worldtube. This restoring force manifests itself as the resistance force which the particle exerts on the ground, i.e. as the inertial force with which the particle resists its acceleration while being at rest on the ground. Therefore it becomes clear that what has been traditionally called the gravitational force acting on the particle, or the particle's weight, is in reality the particle's inertial force with which the particle resists its acceleration when it is at rest on the ground. This explains naturally why "there is no such thing as the force of gravity" in general relativity [11, p. 109].

To summarize, general relativity showed that what has been traditionally called the force of weight of a particle (or the gravitational force acting on a particle) is the inertial force with which the particle resists its acceleration while being at rest on the Earth's surface. As Rindler put it "ironically, instead of explaining inertial forces as gravitational... in the spirit of Mach, Einstein explained gravitational forces as inertial" [16, p. 244]. Indeed, according to Mach the origin of inertia is non-local since he believed that all the masses in the Universe are responsible for the inertial forces (which implies that these forces are gravitational), whereas the now accepted Minkowski's treatment of acceleration in spacetime (as the curvature of an accelerating particle's worldline) implies that inertia is a *local* phenomenon in spacetime since it originates from the *deformation* of an accelerating particle's worldtube. Therefore inertia is not a non-local phenomenon that is caused by the distant masses as Mach argued. One might say that what determines the shape of a free particle's geodesic worldtube (which, when deformed, resists its deformation) are all the masses in the Universe in line with Mach's view. However, that would be misleading since in curved spacetime it is the nearby mass that is essentially responsible for the shape of the geodesics in its vicinity. The shape of the geodesic worldline of a particle falling towards the Earth, for example, is predominantly determined by the Earth's mass and the distant masses have practically no contribution.

Why is the inertial mass equivalent to the gravitational mass? When a particle accelerates, the coefficient of proportionality m_i linking the force and the induced by it acceleration in the equation $\mathbf{F} = m_i\mathbf{a}$ is called the inertial mass of the particle. Since Newton it has been defined as *the measure of the* *resistance a particle offers to its acceleration.* In the Newtonian gravitational theory, when the same particle is at rest on the Earth's surface the coefficient of proportionality m_g linking the force of gravity and the induced by it acceleration in the equation $\mathbf{F} = m_g \mathbf{g}$ is called the (passive) gravitational mass of the particle. Since Newton it has been known that the inertial mass and the gravitational mass are equivalent. But no one knew what this equivalence meant. Einstein merely *postulated* it as another manifestation of the principle of equivalence when he created general relativity.

As we saw above the generalization of Minkowski's representation of accelerated and inertial motion for curved spacetime and taking seriously the reality of particles' worldtubes (and the reality of spacetime itself) naturally explained the equivalence of inertial and (what was called before general relativity) gravitational forces. This effectively also explained the equivalence of inertial and gravitational masses – both masses are inertial. Indeed, whether a particle is accelerating or is on the Earth's surface, in both cases the particle is subject to absolute acceleration (since its worldtube is deformed, i.e., non-geodesic) and the particle resists the change in its inertial motion (i.e., resists the deformation of its worldtube). As the inertial mass is the measure of the resistance a particle offers to its acceleration, it does follow that in both cases the particle's mass is inertial.

Since there have been some recent attempts to deny the reality of the relativistic increase of the mass I think it is appropriate to note that those attempts somehow fail to see the obvious reason for the introduction of relativistic mass – *as inertial mass is the measure of the resistance a body offers to its acceleration and as its acceleration is different in different inertial reference frames, the body's inertial mass cannot be the same in all frames* (for more details see [10, pp. 114-116]).

Are gravitational phenomena caused by gravitational interaction according to general relativity? What follows in this section may seem quite controversial but I think it is worth exploring the implications of general relativity *itself* since the generalization of Minkowski's representation of inertial motion for curved spacetime – the geodesic hypothesis – implies that *gravitational phenomena are not caused by gravitational interaction.* Such a stunning possibility [17] deserves very serious scrutiny because of its implications for fundamental physics as a whole, and particularly for two research programs as mentioned above – detection of gravitational waves and quantum gravity.

As too much is at stake in terms of both the number of physicists working on quantum gravity and on detection of gravitational waves, and the funds being invested in these worldwide efforts, even the heretical option of not taking gravity for granted should be thoroughly analyzed. It should be specifically stressed, however, that such an analysis may require extra effort from relativists who sometimes appear to be more accustomed to solving technical problems than to examining the physical foundation of general relativity which may involve no calculations. Such an analysis is well worth the effort since it ensures that what is calculated is indeed in the proper framework of general relativity and is not smuggled into it to twist it until it yields some features that resemble gravitational interaction.

Had Minkowski lived longer he would have probably been enormously excited to see his profound idea that four-dimensional physics is spacetime geometry so powerfully boosted by Einstein's discovery that gravitation is a manifestation of the non-Euclidean geometry of spacetime. Indeed, the fact that the appearance of gravitational attraction between two free particles arises from the convergence of their geodesic worldlines in curved spacetime is fully in line with Minkowski's anticipation that "the laws of physics can find their most complete expression as interrelations between these worldline" [4, p. 112]. However, keeping in mind how critically and creatively he examined the facts that led to the creation of Einstein's special relativity and how he gave its now accepted spacetime formulation, it is quite reasonable to imagine that Minkowski might have acted in the same way with respect to Einstein's general relativity as well. Imagining such a scenario could help us to examine the logical structure of general relativity by applying the lessons learned from Minkowski's examination of special relativity. Such an examination now seems more than timely especially in light of the fact that the different approaches aimed at creating a theory of quantum gravity [18, 19, 20] have been unsuccessful.

In order to explore rigorously the implications of general relativity *itself* let me state explicitly the following facts from it.

- Like flat (Minkowski) spacetime, the non-Eiclidean spacetime of general relativity is a *static* entity with a forever given network of worldtubes of macroscopic bodies. Relativists are of course aware of this intrinsic feature of spacetime (reflecting its very nature) – that one cannot talk about dynamics in spacetime ("there is no dynamics in spacetime: nothing ever happens there. Spacetime is an unchanging, once-and-for-all picture encompassing past, present, and future" [14, p. 7]). But it seems it is not always easy to regard this counter-intuitive feature of spacetime as adequately representing the world.

- The geometry of spacetime is either intrinsic (pseudo-Euclidean in the case of Minkowski spacetime and pseudo-non-Euclidean in the case of de Sitter's vacuum solution of the Einstein-Hilbert equation) or induced by matter (although it is widely assumed to be clear in general relativity that matter causes the curvature of spacetime, that issue is more subtle than usually presented in the literature as briefly discussed below).

- What is still (misleadingly) called the gravitational field in general relativity is not a physical field; at best, the gravitational field can be regarded as a geometrical field.

- There is no gravitational force in general relativity.

- By the geodesic hypothesis, a timelike geodesic in spacetime represents a free partricle, which moves by inertia.

A close examination of these facts reveals that when general relativity is taken for what it is, it does imply that gravitational phenomena are *fully* explained in the theory without the need to assume that they are caused by gravitational interaction. What has the appearance of gravitational attraction between particles involves only *inertial* (*interaction-free*) motion of *free* particles and is merely a result of the curvature of spacetime. In general relativity falling bodies and the planets are all free bodies which move by inertia and for this reason they do not interact in any way with the Earth and the Sun, respectively, since by its very nature *inertial motion does not involve any interaction.*

I think the major reason for so far missing the opportunity to decode *everything* that general relativity has been telling us about the world is that the existence of gravitational interaction has been taken for granted. As a result of adopting such a fundamental assumption without any critical examination, gravitational interaction has been artificially and forcefully inserted into general relativity through (i) the definition of a free particle (which posits that otherwise free particles are still subject to gravitational interaction), and (ii) the quantity gravitational energy and momentum, which general relativity itself refuses to accommodate.

The often openly stated definition of a free particle in general relativity – a particle is "free from any influences other than the curvature of spacetime" [21] – effectively *postulates* the existence of gravitational interaction by almost explicitly asserting that the influence of the spacetime curvature on the *shape* of a free particle's worldline constitutes gravitational interaction.

To see whether a free particle is subject to gravitational interaction, imagine a wandering planet far away from any galaxy which means that in a huge spacetime region the geometry is close to flat and only the planet's mass induces an observable curvature. Imagine also a free particle in that spacetime region, which travels towards the planet. When far away from the planet, the particle's worldline is straight. But as the particle approaches the planet its worldline becomes increasingly deviated from its straight shape. Despite that its shape changes, the particle's worldline remains geodesic (not deformed) since the curvature of the worldline is simply caused by the spacetime curvature induced by the planet's mass. The standard interpretation of this situation in general relativity, implied by the definition of a free particle, is that the planet, through the spacetime curvature created by its mass, affects the worldline of the particle which is interpreted as gravitational interaction.

However, if carefully analyzed, the assumption that the planet's mass curves spacetime, which in turn changes the shape of the geodesic worldline of a free particle, does not imply that the planet and the particle interact gravitationally. There are four reasons for that.

First, it is assumed that in general relativity the Einstein-Hilbert equation clearly demonstrates that matter determines the geometry of spacetime through the stress-energy of matter T_{ab}. In fact, how that happens (how matter curves spacetime) is the major open question in general relativity. What further complicates the (often taken as self-evident) assertion that matter determines the geometry of spacetime is the fact that in general relativity matter cannot be clearly regarded as something that tells the spacetime geometry how to change since matter itself cannot be defined without that same geometry – "T_{ab} itself is a quantity which refers, not only to "matter", but also to "geometry" " [14, p. 83] (since T_{ab} contains the metric tensor). Therefore, as very little is known of how matter influences the geometry of spacetime, it is unjustified to take for certain that the change of the shape of the worldline of a free particle by the spacetime curvature caused by a massive body constitutes gravitational interaction; moreover, as indicated below the massive body does not spend any additional energy to change the shape of the particle's worldline.

Second, the shape of the geodesic worldlines of free particles is *naturally* determined by the curvature of spacetime which itself may not be necessarily induced by some mass. This is best seen from the fact that general relativity shows *both* that spacetime is curved by the presence of matter, and that a matter-free spacetime can be *intrinsically* curved. The latter option follows from de Sitter's solution [22] of the Einstein-Hilbert equation. Two *test* particles in the de Sitter universe only appear to interact gravitationally since in fact their interaction-like behaviour is caused by the "curvature" of their geodesic worldlines ("curvature" here means non-straightness), which is determined by the *intrinsic* curvature of the de Sitter spacetime. The fact that there are no straight geodesic worldlines in non-Euclidean spacetime (which gives rise to geodesic deviation) manifests itself in the relative acceleration of the test particles towards each other which creates the impression that the particles interact gravitationally. Due to the usual assumption that the masses of *test* particles are negligible in order not to affect the geometry of spacetime, the example with the test particles in the de Sitter universe is a good approximation of a matter-free universe.

Third, the experimental fact that particles of different masses fall towards the Earth with the *same* acceleration in full agreement with general relativity's *"a geodesic is particle-independent"* [12, p. 178], ultimately means that the shape of the geodesic worldline of a free particle in spacetime curved by the presence of matter is determined by the spacetime geometry *alone* and not by the matter. This is best seen from the Einstein-Hilbert equation itself – a body curves *solely* spacetime *irrespective* of whether or not there are other particles there, which means that *no additional energy is spent* for "curving" (not deforming) the geodesic worldlines of any free particles that are in the vicinity of the body. That is why a geodesic is particle-independent. This feature of general relativity taken alone demonstrates that the fact that the shape of the geodesic worldline of a free particle is determined by the curvature of spacetime does not constitute gravitational interaction.

Fourth, if determining the shape of a free particle's geodesic worldline by the spacetime curvature induced by a body's mass-energy constituted gravitational interaction, that would imply some exchange of *gravitational* energy-momentum between the body and the particle. But there is no such a thing as gravitational energy-momentum in general relativity *itself* – its mathematical structure does not allow a proper tensorial expression for a gravitational energy-momentum. This counter-intuitive feature of general relativity is not surprising at all since (i) there is no *physical* gravitational field (one can use the term "field" to describe gravitational phenomena only in the sense of a *geometrical* field, but such a field describes the geometry of spacetime and as such does not possess any energy), and (ii) there is no gravitational force and therefore there is no gravitational energy either since such energy is defined as the work done by gravitational forces.

In short, the mass-energy of a body influences the geometry of spacetime no matter whether or not there are any particles in the body's vicinity, and the shape of a free particle's geodesic worldline reflects the spacetime curvature no matter whether it is intrinsic or induced by matter.

Is there gravitational energy? Although this question was answered above it is necessary to explain briefly why the energy involved in gravitational phenomena is not gravitational. Consider the energy of oceanic tides which is transformed into electrical energy in tidal power stations. The tidal energy is part of gravitational phenomena, but is not gravitational energy. It seems most appropriate to call it *inertial energy* because it originates from the work done by inertial forces acting on the blades of the tidal turbines – the blades further deviate the volumes of water from following their geodesic (inertial) paths (the water volumes are already deviated since they are prevented from falling) and the water volumes *resist* the further change in their inertial motion; that is, the water volumes exert *inertial* forces on the blades. With respect to the resistance, this example is equivalent to the situation in hydroelectric power plants where water falls on the turbine blades from a height (this example is even clearer) – the blades prevent the water from falling (i.e. from moving by inertia) and it resists that change. It is that resistance force (i.e. inertial force) that moves the turbine, which converts the inertial energy of the falling water into electrical energy. According to the standard explanation it is the kinetic energy of the falling water (originating from its potential energy) that is converted into electrical energy. However, it is evident that behind the kinetic energy of the moving water is its inertia (its resistance to its being prevented from falling) – it is the inertial force with which the water acts on the turbine blades when prevented from falling. And it can be immediately seen that the inertial energy of the falling water (the work done by the inertial force on the turbine blades) is equal to its kinetic energy [17, Appendix B].

Do gravitational waves carry gravitational energy? At present there exists a widespread view that there is indirect astrophysical evidence for the existence of gravitational energy. That evidence is believed to come from the interpretation of the decrease of the orbital period of binary pulsar systems, notably the system PSR 1913+16 discovered by Hulse and Taylor in 1974 [23]; recently it was also reported of "evidence for the loss of orbital energy in agreement. . . with the emission of gravitational waves" from a binary system of two candidate black holes [24, 25]. According to this interpretation the decrease of the orbital period of such binary systems is caused by the loss of energy due to gravitational waves emitted by the systems. Almost without being challenged (with only few exceptions [26, 27, 28]) this view holds that quadrupole radiation of gravitational waves which carry gravitational energy away from the binary systems has been indirectly experimentally confirmed.

I think the interpretation that the orbital motion of the neutron stars in the PSR 1913+16 system, for example, loses energy by emission of gravitational waves should be rigorously reexamined since it *contradicts general relativity*, particularly the geodesic hypothesis and the experimental evidence which confirmed it. The reason is that by the geodesic hypothesis the neutron stars, whose worldlines had been regarded as exact *geodesics* (since the stars had been "modelled dynamically as a pair of orbiting point masses" [29]), *move by inertia without losing energy* because the very essence of inertial motion is motion without any loss of energy. For this reason no energy can be carried away by the gravitational waves emitted by the binary pulsar system. Therefore the experimental fact of the decay of the orbital motion of PSR 1913+16 (the shrinking of the stars' orbits) cannot be regarded as evidence for the existence of gravitational energy. The observed diminishing of the orbital period of the binary pulsar should be caused by other mechanisms, e.g. magnetic or (and) tidal effects. Tidal friction was suggested in 1976 [30] as an alternative to the explanation given by Hulse and Taylor, which ignored the tidal effects by treating the neutron stars as point masses. The argument that the neutron stars would behave as rigid bodies (since they are believed to be very compact) is not convincing because by the same reason – the large spacetime curvature caused by the stars (which is ultimately responsible for their rigidity) – the other gravitational effects, i.e., the tidal effects, are also very strong.

If it really turns out that binary pulsars are not slowed by the emission of gravitational energy (as I believe it would), that would be another important lesson of the superior role of physics over mathematics in physical theories. Being aware that not devoting particular attention to physical (conceptual) analyses of physical situations could lead to problems [32], Wheeler stressed that the superiority of physics should always and explicitly be kept in mind in what he called the first moral principle [31]:

> Never make a calculation until you know the answer. Make an estimate before every calculation, try a simple physical argument (symmetry! invariance! conservation!) before every derivation, guess the answer to every paradox and puzzle.

In the case of the decrease of the orbital period of binary systems, the physical argument is that the geodesic hypothesis and the statement that bodies, whose worldlines are *geodesic*, emit gravitational energy cannot be both correct. Another physical argument in the case of binary systems involves orbital energy. Saying that a binary system of two neutron stars has orbital (gravitational) energy is equivalent to saying that two bodies in uniform relative motion approach each other in flat spacetime also have some common energy since in both cases only inertial motion is involved – the stars' worldlines are geodesic in curved spacetime and the bodies worldlines are straight in flat spacetime.

The two cases are equivalent since the stars also move by inertia and there is no exchange of some gravitational energy between them – as discussed above the same stress-energy tensor of each star produces the same spacetime curvature no matter whether or not the other star is there.

Can gravity be quantized? In the case of physical interactions when one talks about the energy associated with an interaction, it is the energy of the entity (the field and its quanta) that mediates an interaction and it is that entity and its energy which are quantized. What should make us to consider seriously the possibility that a theory of quantum gravity might be impossible is the fact that there is no such a thing as an entity which mediates gravitational interaction in general relativity. Although the term "gravitational field" is widely used in the general relativistic literature its correct meaning is to describe the geometry of spacetime and nothing more. It is not a physical field that can be quantized. If the gravitational field represented some physical entity, it should be measurable. Misner, Thorne, and Wheeler paid special attention to the question of the measurement of the gravitational field [33, p. 399]:

> "I know how to measure the electromagnetic field using test charges; what is the analogous procedure for measuring the gravitational field?" This question has, at the same time, many answers and none.
>
> It has no answers because nowhere has a precise definition of the term "gravitational field" been given – nor will one be given. Many different mathematical entities are associated with gravitation: the metric, the Riemann curvature tensor, the Ricci curvature tensor, the curvature scalar, the covariant derivative, the connection coefficients, etc. Each of these plays an important role in gravitation theory, and none is so much more central than the others that it deserves the name "gravitational field." Thus it is that throughout this book the terms "gravitational field" and "gravity" refer in a vague, collective sort of way to all of these entities. Another, equivalent term used for them is the "geometry of spacetime."

As there is no physical entity which is represented by the term "gravitational field" in general relativity it does follow that there is no energy and momentum of that non-existent physical entity. This in turn should make us to accept the unambiguous fact that the logical structure of general relativity does not contain and does not allow a *tensor* of the gravitational energy and momentum. It was Einstein who first tried to insert the concept of gravitational energy and momentum *forcefully* into general relativity (since he represented it by a pseudotensor, not a tensor as it should be) in order to ensure that gravity can still be regarded as some interaction. Einstein made the gigantic step towards the profound understanding of gravity as spacetime curvature but even he seems to have been unable to accept all implications of the revolutionary view of gravitational phenomena.

For decades the efforts of many brilliant physicists to create a quantum theory of gravity have not been successful. This could be an indication that those efforts might not have been in the right direction. In such desperate times in fundamental physics all approaches and ideas should be on the research table, including the approach discussed here – that general relativity completely explains gravitational phenomena without the need of gravitational interaction, if gravity is consistently and rigorously regarded as a manifestation of the non-Euclidean geometry of spacetime, that is, general relativity implies that gravitational phenomena are not caused by gravitational interaction. An immediate implication of this approach is that quantum gravity understood as quantization of gravitational interaction is impossible because there is nothing to quantize. If this turns out to be the case, the efforts to quantize the apparent gravitational interaction should be redirected towards what seems to be the actual open question in gravitational physics – how matter curves spacetime – since it is quantum physics which should deal with this question and which should provide the definite answer to the central question in general relativity – whether or not there exists some kind of interaction between physical bodies mediated by spacetime itself.

3 Propagation of light in non-inertial reference frames in spacetime

So far the issue of the propagation of light in non-inertial reference frames (accelerating in special relativity and associated with a body in general relativity whose worldline is not geodesic) has not been fully presented in the books on relativity (I am aware only of two books where only the slowing down of light in curved spacetime is explicitly discussed [34, 35]). This issue has a straightforward and self-evident explanation when the physical phenomenon of propagation of light is regarded as spacetime geometry. In fact, regarding the phenomenon of light propagation as spacetime geometry naturally explains both why the speed of light is the same in all inertial reference frames in flat spacetime and why it is not constant in non-inertial reference frames (in flat and curved spacetimes).

Let us start with the propagation of light in inertial reference frames. In his 1905 paper Einstein merely postulated (as his second postulate) that the speed of light is the same in all inertial frames. It is clear now that Einstein did not need to introduce a second postulate in special relativity since the constancy of the speed of light follows from the first postulate (the principle of relativity) – the consequence of Maxwell's equations that electromagnetic waves propagate with a constant speed (which turned out to be a fundamental constant $c = (\varepsilon_0\mu_0)^{-1/2}$) should hold in all inertial frames. However, at the time when Michelson and Morley proved experimentally that the speed of light is constant and a bit later when Einstein postulated it, that fact had been a complete mystery.

The situation completely changed in 1908 when Minkowski gave the four-dimensional formulation of special relativity. One of the implications of Minkowski's four-dimensional physics was the explanation of the constancy of the speed of light in inertial frames. In the ordinary "three-dimensional" (space *and* time) language, the speed of light is the same in all inertial frames because each reference frame has not only its own (proper) time, but also (as Minkowski showed) its own (proper) space and light propagates with respect to the proper space of each frame and the frame's proper time measures the duration of the light propagation.

However, complete understanding of the whole phenomenon of propagation of light is obtained when the physics of this phenomenon is regarded as spacetime geometry. Only when Minkowski gave the spacetime formulation of special relativity it was revealed that there are three kinds of length in spacetme and that the propagation of light is represented by null (or lightlike) geodesics whose status is absolute or frame independent. A light signal which travels the distance $\mathrm{d}x$ for the time period $\mathrm{d}t$ in *any* inertial reference frame is represented in flat spacetime by a lightlike worldline whose length between the events of emission and arrival of the light signal is zero (in the case of a two-dimensional spacetime):

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - \mathrm{d}x^2 = 0\,.$$

It is evident from here that in any inertial reference frame the speed of light is the same: $c = \mathrm{d}x/\mathrm{d}t$.

However, even in flat spacetime the spacetime metric in a non-inertial reference frame (e.g., an elevator accelerating with a proper acceleration a along the x-axis) is [33, p. 173]:

$$\mathrm{d}s^2 = \left(1 + \frac{ax}{c^2}\right)^2 c^2\,\mathrm{d}t^2 - \mathrm{d}x^2\,. \tag{1}$$

It is immediately seen from here that for a lightlike worldline (representing a propagating light signal) $\mathrm{d}s^2 = 0$ and therefore the coordinate anisotropic velocity of light c^{a} at a point x is

$$c^{\mathrm{a}}(x) = \pm c\left(1 + \frac{ax}{c^2}\right)\,, \tag{2}$$

where the $+$ and $-$ signs correspond to the propagation of a light signal along or against the x-axis, respectively.

As spacetime is flat it is clear that the non-constancy of the velocity of light in an accelerating elevator is not caused by the curvature of spacetime. It is seen from (1) that the non-Euclidean metric in the accelerating elevator results from the curvature of the elevator's worldline along which the time axis is constantly chosen (at each point of the elevator's worldline the time axis is the tangent at that point and coincides with the time axis of the instantaneously comoving inertial reference frame at that point). In 1960 Synge stressed the need to distinguish between two types of effects in relativity [11]: "Space-time is either flat or curved, and in several places in the book I have been at considerable pains to separate truly gravitational effects due to curvature of space-time from those due to curvature of the observer's world-line (in most ordinary cases the latter predominate)." The anisotropic velocity of light (2) is another manifestation of the latter effect.

That the velocity of light is not constant in an accelerating elevator was first realized by Einstein whose thought experiments involving an accelerating elevator and an elevator at rest on the Earth's surface led him to the discovery that a horizontal light signal bends in such elevators (as shown in Fig. 3) [36]: "A curvature of rays of light can only take place when the velocity of propagation of light varies with position." The implications of this results have not been fully explored. Although the bending of a horizontal light ray in Einstein's original thought experiments with elevators found their way even in introductory physics textbooks [37, 38, 39, 40], the obvious question of whether light rays propagating in a vertical direction (parallel and anti-parallel to the elevator's acceleration) are also affected by the elevators's acceleration, has never been asked. The definite answer to this question could have been given even before Minkowski's spacetime formulation of special relativity.

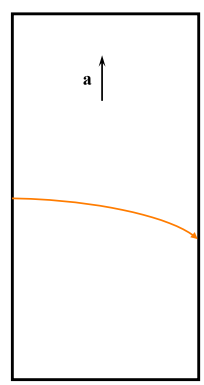

Figure 3: A horizontal light ray propagates in an accelerating elevator

Consider an inertial reference frame I in which an elevator is at rest. At a given moment t_0 the elevator starts to accelerate upwards as shown in Fig. 4 [10, Sect. 7.3]. The x-axes of I and a non-inertial frame N associated with the elevator are along the elevator's acceleration. At the same moment t_0 three light rays are emitted *simultaneously* in the elevator from points D, A, and C towards point B. As at that moment I and N are at rest the emission of the light rays is simultaneous in I as well (now we can say that I is the instantaneously comoving inertial frame at the moment t_0 which means that I and N share the same instantaneous space and therefore they share the same class of simultaneous events at t_0).

At the next moment as N accelerates an observer in I sees that the three light rays arrive simultaneously not at point B, but at B' (since during the time the light rays travel the elevator, i.e., N, moves upwards); the inertial observer sees that the horizontal light ray emitted from D propagates along a straight line (the dashed yellow line in Fig. 4). Let $DB = AB = BC = r$ in I. Since

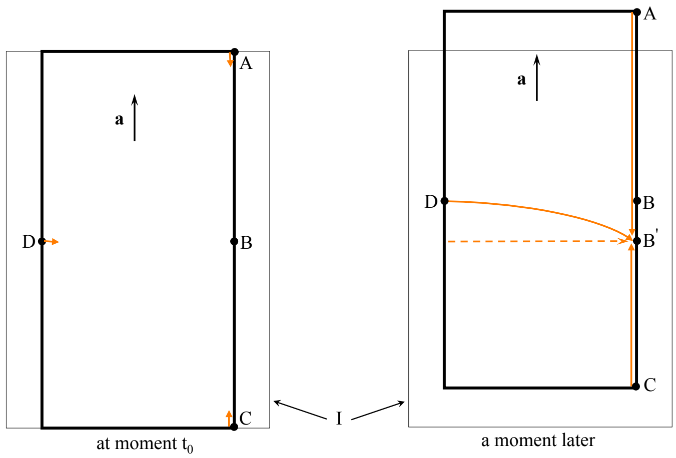

Figure 4: How an inertial observer I and an observer in an accelerating elevator see the propagation of three light rays in the elevator.

for the time $t = r/c$ in I the light rays travel towards B, the elevator moves a distance $\delta = at^2/2 = ar^2/2c^2$. As the simultaneous arrival of the three rays at point B' as viewed in I is an absolute (observer-independent) fact due to its being a *single* event, it follows that the rays arrive simultaneously at B' as seen by an observer in N as well.

We have $DB = AB = BC = r$ in both I and N because this thought experiment represents a clearly non-relativistic situation and therefore the relativistic contraction of AB and BC in I can safely be ignored (the elevator just started to accelerate and its velocity relative to I is negligible compared to c). Since for the *same* coordinate time $t = r/c$ in N, the three light rays travel *different* distances $DB' \approx r$, $AB' = r+\delta$, and $CB' = r-\delta$, before arriving *simultaneously* at point B', an observer in the elevator concludes that the propagation of light is affected by the elevator's acceleration. The *average* velocity $c^{\mathrm{a}}_{AB'}$ of the light ray propagating from A to B' is slightly greater than c:

$$c^{\mathrm{a}}_{AB'} = \frac{r+\delta}{t} \approx c\left(1 + \frac{ar}{2c^2}\right) .$$

The average velocity $c^{\mathrm{a}}_{B'C}$ of the light ray propagating from C to B' is slightly smaller than c:

$$c^{\mathrm{a}}_{CB'} = \frac{r-\delta}{t} \approx c\left(1 - \frac{ar}{2c^2}\right) .$$

It is easily seen that to within terms proportional to c^{-2} the average light velocity between A and B is equal to that between A and B', i.e., $c^{\mathrm{a}}_{AB} = c^{\mathrm{a}}_{AB'}$ and also $c^{\mathrm{a}}_{CB} = c^{\mathrm{a}}_{CB'}$:

$$c^{\mathrm{a}}_{AB} = \frac{r}{t-\delta/c} = \frac{r}{t - at^2/2c} = \frac{c}{1 - ar/2c^2} \approx c\left(1 + \frac{ar}{2c^2}\right) \qquad (3)$$

and

$$c^{\mathrm{a}}_{CB} = \frac{r}{t + \delta/c} \approx c\left(1 - \frac{ar}{2c^2}\right). \tag{4}$$

Since the *coordinate* time t is involved in the calculation of the average velocities (3) and (4), it is clear that these expressions represent the average *coordinate* velocities between the points A and B and the points C and B, respectively.

The same expressions for the average coordinate velocities c^{a}_{AB} and c^{a}_{CB} can also be obtained from the expression for the coordinate velocity of light (2) in N. As the coordinate velocity $c^{\mathrm{a}}(x)$ is continuous on the interval $[x_A, x_B]$, one can calculate the average coordinate velocity between A and B in Fig. 4:

$$c^{\mathrm{a}}_{AB} = \frac{1}{x_B - x_A}\int_{x_A}^{x_B} c^{\mathrm{a}}(x)\,\mathrm{d}x = c\left(1 + \frac{ax_B}{c^2} + \frac{ar}{2c^2}\right), \tag{5}$$

where we have taken into account the fact that $x_A = x_B + r$. When the coordinate origin is at point B ($x_B = 0$), the expression (5) coincides with (3). In the same way,

$$c^{\mathrm{a}}_{BC} = c\left(1 + \frac{ax_B}{c^2} - \frac{ar}{2c^2}\right), \tag{6}$$

where $z_C = x_B - r$. For $x_B = 0$, (6) coincides with (4).

Analogous expressions can be obtained for the average coordinate velocity of light in an elevator at rest on the Earth's surface, which is subject to the acceleration due to gravity g [10, Sect. 7.3]:

$$c^{\mathrm{g}}_{AB} = c\left(1 + \frac{gx_B}{c^2} + \frac{gr}{2c^2}\right) \tag{7}$$

and

$$c^{\mathrm{g}}_{BC} = c\left(1 + \frac{gx_B}{c^2} - \frac{gr}{2c^2}\right). \tag{8}$$

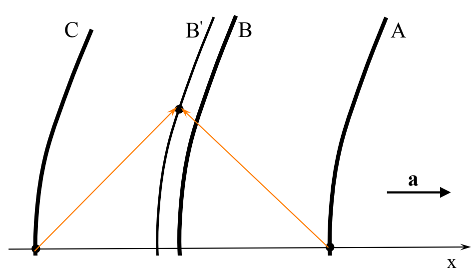

Figure 5: Regarding the physical phenomenon of light propagation as spacetime geometry provides a straightforward explanation of the anizotropic propagation of light in the accelerating elevator – the non-constancy of the velocity of light observerd in the elevator is caused by the curvature of the worldline of point B

As indicated above representing the physical situation depicted in Fig. 4 in terms of spacetime geometry is the best way to demonstrate that it is the *curvature* of the worldline of point B (and B') which causes the non-constancy of the velocity of light in N. This is done in Fig. 5 which represents a two-dimensional spacetime diagram. The worldlines of points A, B, B', and C as well as the worldlines of the light rays emitted from A and C are depicted in the figure. It is obvious that due to the curvature of the worldline of B (and B') the worldlines of the light rays meet at the worldline of B', not at worldline of B. In this thought experiment it is the curvature of the worldline B alone which is responsible for the anisotropic velocity of light in the accelerating elevator, but in more complex experiments with light rays in an accelerating elevator the curvature of the worldlines of the light sources and the light detectors causes the anisotropy in the propagation of light in non-inertial reference frames.

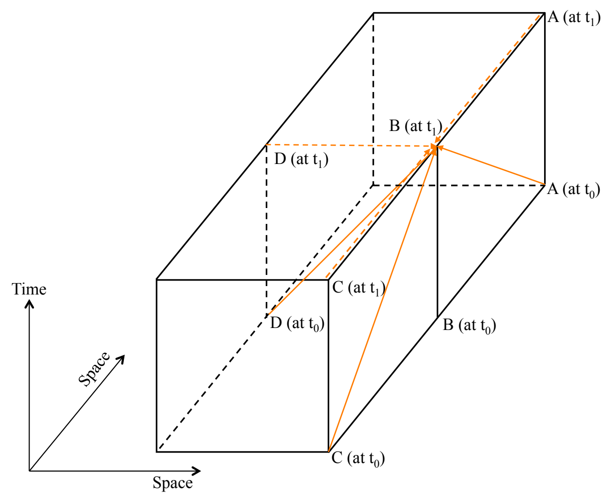

Figure 6: The spacetime geometry of the propagation of three light rays in an inertial elevator

The spacetime geometry of the propagation of all three light rays emitted from points A, C, and D can be represented in a three-dimensional spacetime diagram. In order to make the spacetime diagram of the accelerating elevator more easily understandable, let us consider first the propagation of the three light rays in an elevator, which moves with constant velocity as shown in Fig. 6. The elevator at the moment t_0, when the three light rays are emitted simultaneously towards point B, is represented by the bottom side of the parallelepiped in Fig. 6. At moment t_1, when the worldlines of the three light rays meet at the worldline of point B, the elevator is represented by the top side of the parallelepiped.

Now consider the spacetime diagram in Fig. 7 showing the propagation of the three light rays in an accelerating elevator. The elevator at moments t_0 and t_1 is represented by the bottom and top sides of the parallelepiped, respectively (the two sides of the parallelepiped represent the instantaneous spaces of the non-inertial reference associated with the elevator, which correspond to the moments t_0 and t_1). It is again quite obvious that what causes the anisotropic propagation of light in the accelerating elevator (in this specific thought experiment) is the curvature of the worldline of point B – the worldlines of the light rays emitted from A, C and D at t_0 all meet at the worldline of point B'.

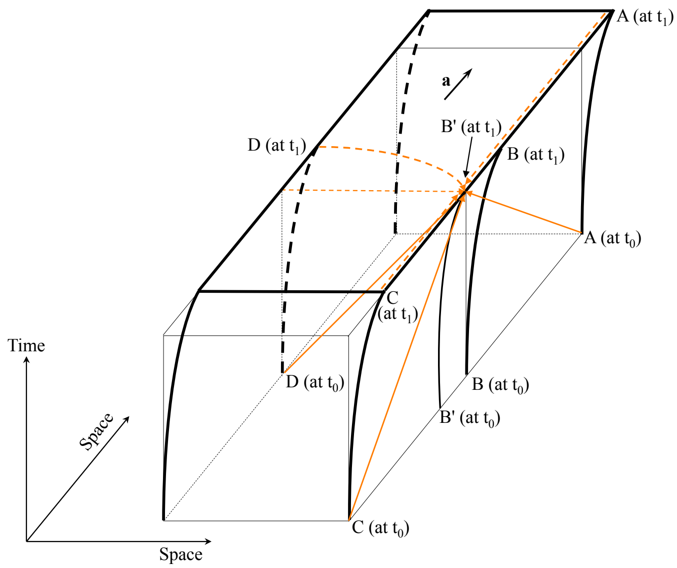

Figure 7: The spacetime geometry of the propagation of three light rays in an accelerating elevator. In order not to complicate the spacetime diagram, the elevator at moments t_0 and t_1 is shown as the bottom and top sides of the parallelepiped. However, in reality the two sides (representing the elevator at t_0 and t_1 and also the instantaneous spaces at t_0 and t_1 of the non-inertial reference frame N associated with the elevator) are not parallel, because those sides (i.e. the instantaneous spaces of N at the two moments) coincide with the spaces of the instantaneously comoving inertial reference frames at t_0 and t_1, which are not paralel since the two instantaneously comoving inertial reference frames are in relative motion.

It turns out that the average coordinate velocity of light is not sufficient for the complete description of propagation of light in non-inertial reference frames. The average coordinate velocity of light explains the propagation of light in such frames in situations like the one discussed above. However, in a situation where the average light velocity between two points – a source and an observation point – is determined *with respect to one of the points*, where the local velocity of light is c and where the *proper time* is used, that average velocity of light is not coordinate; it can be regarded as an average *proper* velocity of light. For instance, such a situation occurs in the Shapiro time delay effect.

We calculated the average coordinate velocity of light in an accelerating elevator, but now we will determine the average proper velocity of light in a non-inertial reference frame N associated with an elevator at rest on the

Earth's surface. The reason is to explain in detail how light propagates towards and away from the Earth since this issue is not always explained properly in introductory physics textbooks. For example, one can read that "a beam of light will accelerate in a gravitational field, just like objects that have mass" and therefore "near the surface of the earth, light will fall with an acceleration of 9.81 m/s^2" [37]. We shall now see that during its "fall' towards the Earth, light is slowing down – a negative acceleration of 9.81 m/s^2 is decreasing its velocity.

As an elevator at rest on the Earth is prevented from falling it is accelerating (since its worldtube is curved) with an acceleration g due to gravity.

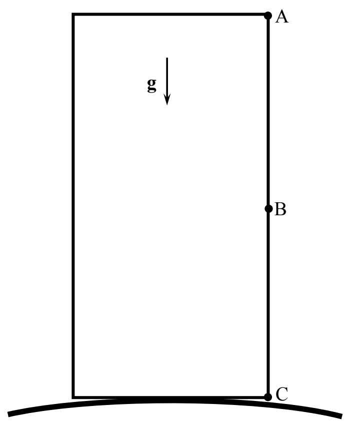

Figure 8: An elevator at rest on the Earth's surface

To calculate the average proper velocity of light which originates from B and is observed at A, we have to determine the initial velocity of a light signal at B and its final velocity at A, both with respect to A [10, Sect. 7.4]. As the local velocity of light is c, the final velocity of the light signal determined at A is obviously c. By taking into account that in a parallel "gravitational field," proper and coordinate distances are the same [41], we can determine the initial velocity of the light signal at B as seen from A:

$$c_B^{\mathrm{g}} = \frac{\mathrm{d}x_B}{\mathrm{d}\tau_A} = \frac{\mathrm{d}x_B}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}\tau_A} \ .$$

Here $\mathrm{d}\tau_A = \mathrm{d}s_A/c$ is the proper time of an observer with constant spatial coordinates at A,

$$\mathrm{d}\tau_A = \left(1 + \frac{gx_A}{c^2}\right)\mathrm{d}t \ ,$$

and $\mathrm{d}x_B/\mathrm{d}t = c^{\mathrm{g}}(x_B)$ is the coordinate velocity of light at B,

$$c^{\mathrm{g}}(x_B) = c\left(1 + \frac{gx_B}{c^2}\right) \ ,$$

which follows from the metric (the line element) in the case of parallel "gravitational field" [33, p. 1056]:

$$\mathrm{d}s^2 = \left(1 + \frac{gx}{c^2}\right)^2 c^2\,\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 \,.$$

As $x_A = x_B + r$ (we again have $AB = BC = r$) and $gx_A/c^2 < 1$ (since for any value of x in N, there exists the restriction $|x| < c^2/g$), for the coordinate time $\mathrm{d}t$, we have (to within terms $\propto c^{-2}$)

$$\mathrm{d}t \approx \left(1 - \frac{gx_A}{c^2}\right) \mathrm{d}\tau_A = \left(1 - \frac{gx_B}{c^2} - \frac{gr}{c^2}\right) \mathrm{d}\tau_A \;.$$

Then for the initial velocity c_B^{g} at B as determined at A, we obtain

$$c_B^{\mathrm{g}} = c\left(1 + \frac{gx_B}{c^2}\right)\left(1 - \frac{gx_B}{c^2} - \frac{gr}{c^2}\right) \;,$$

or, keeping only the terms proportional to c^{-2},

$$c_B^{\mathrm{g}} = c\left(1 - \frac{gr}{c^2}\right) \;. \tag{9}$$

Therefore an observer at A will determine that when a light signal is emitted at B with the initial velocity (9) during the time of its journey towards A (away from the Earth's surface) it will *accelerate* with an acceleration g and will arrive at A with a final velocity equal to c.

For the average proper velocity $\bar{c}_{BA}^{\mathrm{g}} = (1/2)(c_B^{\mathrm{g}} + c)$ of light propagating from B to A as observed at A, we have

$$\bar{c}_{BA}^{\mathrm{g}}(\text{as observed at } A) = c\left(1 - \frac{gr}{2c^2}\right) \;. \tag{10}$$

As the local velocity of light at A (measured at A) is c, it follows that if a light signal propagates from A towards B, its initial velocity at A is c, the final velocity of the light signal at B is (9) and therefore, as seen from A, it is subject to a negative acceleration g and will *slow down* as it 'falls' towards the Earth. The average proper velocity $\bar{c}_{AB}^{\mathrm{g}}$ (as seen from A) of a light signal emitted at A with the initial velocity c and arriving at B with the final velocity (9) will be equal to the average proper velocity $\bar{c}_{BA}^{\mathrm{g}}$ (as seen from A) of a light signal propagating from B toward A. Thus, as seen from A, the back and forth average proper speeds of light travelling between A and B are the *same*.

Now let us determine the average proper velocity of light between B and A with respect to point B. A light signal emitted at B as seen from B will have an initial (local) velocity c there. The final velocity of the signal at A as seen from B will be

$$c_A^{\mathrm{g}} = \frac{\mathrm{d}x_A}{\mathrm{d}\tau_B} = \frac{\mathrm{d}x_A}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}\tau_B} \;,$$

where $\mathrm{d}x_A/\mathrm{d}t = c^{\mathrm{g}}(x_A)$ is the coordinate velocity of light at A,

$$c^{\mathrm{g}}(x_A) = c\left(1 + \frac{gx_A}{c^2}\right) \;,$$

and $d\tau_B$ is the proper time at B,

$$d\tau_B = \left(1 + \frac{gx_B}{c^2}\right) dt \ .$$

Then as $x_A = x_B + r$, we obtain for the velocity of light at A, as determined at B,

$$c_A^{\mathrm{g}} = c\left(1 + \frac{gr}{c^2}\right) \ . \tag{11}$$

Using (11), the average proper velocity of light propagating from B to A as determined from B becomes

$$\vec{c}_{BA}^{\mathrm{g}}(\text{as observed at } B) = c\left(1 + \frac{gr}{2c^2}\right) \ . \tag{12}$$

If a light signal propagates from A to B, its average proper velocity $\vec{c}_{AB}^{\mathrm{g}}$ (as seen from B) will be equal to $\vec{c}_{BA}^{\mathrm{g}}$ (as seen from B) – the average proper speed of light propagating from B to A. This demonstrates that, for an observer at B, a light signal emitted from B with velocity c will *accelerate* toward A with an acceleration g and will arrive there with the final velocity (11). As determined by the B-observer, a light signal emitted from A with initial velocity (11) will be *slowing down* (with $-g$) as it 'falls' towards the Earth and will arrive at B with a final velocity equal to c. Therefore an observer at B will agree with an observer at A that a light signal will *accelerate* with an acceleration g on its way from B to A and will *decelerate* while 'falling' towards the Earth during its propagation from A to B, but will disagree on the velocity of light at the points A and B.

The use of the average anisotropic velocity of light in the Shapiro time delay and the Sagnac effect is demostrated in [10, Sects. 7.5, 7.8].

The calculation of the average proper velocity of light in an accelerating frame is obtain in the same way and gives [10, Sect. 7.4]:

$$c_{BA}^{\mathrm{a}}\,(\text{as observed at } A) = c\left(1 - \frac{ar}{2c^2}\right) \tag{13}$$

and

$$c_{BA}^{\mathrm{a}}\,(\text{as observed at } B) = c\left(1 + \frac{ar}{2c^2}\right), \tag{14}$$

where a is the proper acceleration of the frame.

Comparing the average coordinate velocities of light (5) and (6) with (7) and (8) and the average proper velocities of light (13) and (14) with (10) and (12) shows that their expressions are the same in an accelerating elevator and in an elevator on the Earth's surface. This fact can be regarded as another manifestation of the equivalence principle. But this principle only *postulates* such equivalences without any explanation; they are pure mystery. The complete explanation of the identical anisotropy in the propagation of light in both elevators is obtained only when the phenomenon of propagation of light is regarded as geometry of a real spacetime. Only then it becomes clear that acceleration is a curvature of a worldline. Only then it becomes clear that, like an accelerating elevator, an elevator on the Earth's surface also accelerates since its worldtube, like the worldtube of the accelerating elevator, is also *curved*. Then the same accelerations $a = g$ of the elevators demonstrate that their worldtubes are equally curved, which causes the identical anisotropic propagation of light in an accelerating elevator and in an elevator at rest on the Earth's surface. The fact that the worldlines of the points of the accelerating elevator are as much deviated from their geodesic shapes (i.e., from their straight shapes in flat spacetime) as the worldlines of the points of the elevator on the Earth's surface are deviated from their geodesic shapes in curved spacetime naturally explains the equivalence of all physical phenomena in the elevators (which equivalence was postulated as the equivalence principle).

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